Train Seat Planning Using Queuing TheoryModels of Railway Reservation Occupancy: A Sensitivity Analysis

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ABSTRACT

Long-range planning for Railway seats is becoming an increasingly common phenomenon. However, the tools available for carrying out such planning are not entirely satisfactory. This paper focuses on models for determining how many acute seats are needed to give adequate service to a senior citizen and emergency cases without undue delay. Although this side steps the issue of appropriateness of utilization (by requiring that the issue be settled before specifying the forecasted demands), the problem is still difficult. Involved is a trade-off between low average seat occupancy and shortages of seats due to unusually high demand especially in seasons. ? This paper represents a beginning of an investigation of which details are important, with the ultimate goal of specifying the simplest model possible for use in long-range seat occupancy planning. Accordingly, the paper refers only to a few of the simplest models, ignoring many important articles on forecasting and scheduling seat. The general method and the primary references are discussed in the first section, listing the assumptions that are to be tested. A new “special case relationship” between two models is noted. The second section discusses some of the statistical problems encountered in the use of simulation, and explains the methods of significance testing used here. The third section reports the results of a series of simulation experiments designed to test the assumptions, and the last section discusses the results, directions for additional work, and some anticipated difficulties.
Keywords: Queuing theory, Queuing models, Basic Queuing Process, Queuing Simulation

INTRODUCTION

Jayeshkumar[1], Chadhaury[2] and Patel[3] explain the review of queuing theory and for empirical study the Ticket windows service unit of **Railway Station** is chosen as an example:

The main purpose of this paper is to review the application of queuing theory and to evaluate the parameters involved in the service unit for the Ticket window in **Railway Station**. Therefore, a mathematical model is developed to analyze the performance of the checking out service unit. Two important results need to be known from the data collected in the **Railway Station** by the mathematical model: one is the ‘service rate’ provided to the Passenger during the purchasing ticket process, and the other is the gaps between the arrival times (interval time) of each Passenger per hour. There are **n in e-**ticket window in **Railway Station**, which means consisting of nine servers with nine queues in terms of Queuing Theory. A queue forms whenever current demand exceeds the existing capacity to serve when each window counter is so busy that arriving Passengers cannot receive immediate ticket. So each server process is done as a queuing model in this situation. The data used in the Queuing model is collected for an arrival time of each Passenger in one week by observing. The observations for number of Passengers in a queue, their arrival-time and departure-time were taken without distracting the employees of

Railway station. The whole procedure of the service uniteach day was observed and recorded using a time-watch during the same time period for each day. The aim of studying queuing system simulation is trying to detect the variability in a quality of service due to queues in Ticket window service units, find the average queue length before getting served in order to improve the quality of the services where required, and obtain a sample performance result to obtain time-dependent solutions for complex queuing models. The defined model for this kind of situation where a network of queues is formed is time-dependent and needs to run simulation. The results obtained from **Railway station (Ahemdabad)** using queuing model suggest applying other Railway station of big cities.

In this paper we investigate the important details, with the ultimate goal of specifying the simplest model possible for use in long-range seat occupancy planning and also refer the simplest models, ignoring many important articles on forecasting and scheduling seat. In this paper a new "special case relationship" between two models is noted. Further we also discuss some of the statistical problems encountered in the use of simulation, and explain the methods of significance testing used here. We use the results of a series of simulation experiments designed to test the assumptions, and also directions for additional work, and some anticipated difficulties.

QUEUING THEORY

Delays and queuing problems are most common features not only in our daily-life situations such as at a bank or postal office, at a ticketing office, in public transportation or in a traffic jam but also in more technical environments, such as in manufacturing, computer networking and telecommunications. They play an essential role for business process re-engineering purposes in administrative tasks. "Queuing models provide the analyst with a powerful tool for designing and evaluating the performance of queuing systems." (Bank, Carson, Nelson & Nicol, 2001)Whenever Passengers arrive at a service facility, some of them have to wait before they receive the desired service. It means that the Passenger has to wait for his/her turn, may be in a line. Passengers arrive at a service facility (sales checkout zone Big Bazar) with several queues, each with one server (sales checkout counter). The Passengers choose a queue of a server according to some mechanism (e.g., shortest queue or shortest workload). Sometimes, insufficiencies in services also occur due to an undue wait in service may be because of new employee. A delay in service jobs beyond their due time may result in losing future business opportunities. Queuing theory is the study of waiting in all these various situations. It uses queuing models to represent the various types of queuing systems that arise in practice. The models enable finding an appropriate balance between the cost of service and the amount of waiting.

QUEUING MODELS WITH SINGLE STAGE (FACILITY)

The term queuing system is used to indicate a collection of one or more waiting lines along with a server or collection of servers that provide service to these waiting lines. The example of **Railway Station** is taken for queuing system discussed in this section include:

1) A single waiting line and multiple servers (fig.1),
2) Multiple waiting lines (arranged by priority) and multiple servers (fig.2) and
3) A single waiting line and a single server (fig.3).

All results are presented in next chapter assuming that FIFO is the queuing discipline in all waiting lines and the behavior of queues is **jockey**[1].The **Railway Station** consists of multiple units of ticket window, each unit contains one employee. This kind of a system is called a multiple-server system with single service facility, in other words multiple ticket windows (service units) as a server available in a system. There are two possible models for multiple-server system: Single-Queue Multiple-Server model, and Multiple-Queue Multiple-Server model. Using the same concept of model, the ticket window units are all together taken as a series of servers that forms either single queue or multiple queues for tickets.(single service facility) where the arrival rate of Passengers in a queuing system and service rate per busy server are constants regardless of the state of the system (busy or idle). For such a model the following assumptions are made:

ASSUMPTIONS

According to Jayeshkumar[1], Chadhaury[2] and Patel[3:]

a) Arrivals of Passengers follow a Poisson process

i) The number of Passengers that come to the queue of ticket window server during time period [t, t+s) only depends on the length of the time period ‘s’ but no relationship with the start time ‘t’.

ii) If s is small enough, there will be at most one Passenger arrives in a queue of a server during time period [t, t+s). Therefore, the number of Passengers that arrive in an interval [t, t+s) follows a Poisson distribution and the arrivals of them in a queue follows a Poisson process.

A Poisson process as a sequence of events ‘randomly spaced in time’. The Passenger enters one line and then switches to a shorter line to reduce the waiting time.

b) Inter arrival times of a Poisson process are exponentially distributed

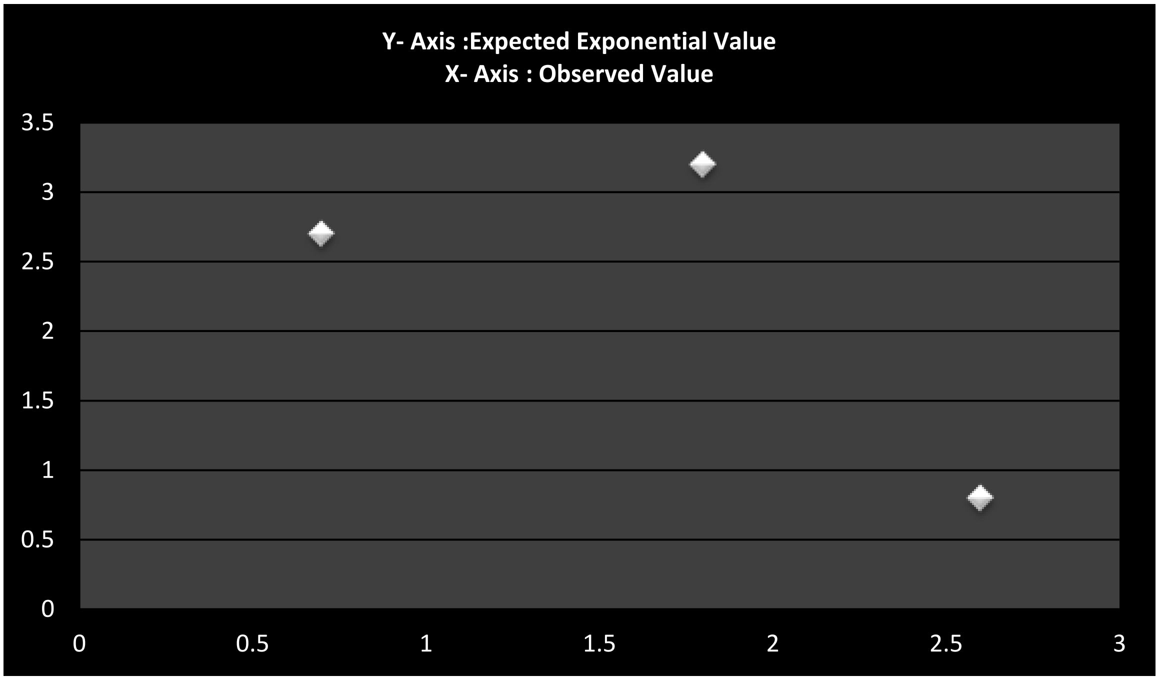
Let $T_1 =$ the time until the next arrival fromt_0 to t_1 $i.e. (t_0 - t_1)$

and $P(T_1 > t) = P_0(t) = e^{-\mu t}$ then $P(T_1 \leq t) = F_{T_1}(t) = 1 - e^{-\mu t}$ and $f_{T_1}(t) = \mu e^{-\mu t}$ for $t > 0$

Similarly the random variables $T_1, T_2, T_3, \ldots T_n, \ldots$ of interarrival times are independent of each other and each has an exponential distribution with mean $1/_{\mu}$.

c) Service times are exponentially distributed

This has been examined by Q-Q plot of collected data given below. The length of the time between arrivals and departures contain the length of the queue and the service time. So the service times are exponentially distributed. Q-Q plot shows the service time is exponentially distributed: Exponential Q-Q Plot of Service Time



and there is one more thing to mention is that there are only a few points on the graph but the number of observations in the original data is nearly 100. The reason for this condition is that, the data was not observed per seconds, whereas service may vary per second. Therefore, some service time has identical value of time.

d) Identical service facilities (same sales checkout service on each server)

e) No Passenger leaves the queue without being served.

f) Infinite number of Passengers in queuing system of ICA (i.e. no limit for queue capacity).

g) FIFO (First in First Out) or FCFS (First Come First Serve)

Passengers arriving from different flows are treated equally by placing into the queues, respecting strictly, their arriving order. Already in the queue are served in the same order they entered, this means, first Passenger that comes in the queue is the first one that goes out. All Passengers arriving in the queuing system will be served approximately equally distributed service time and being served in an order of first come first serve, whereas Passenger choose a queue randomly, or choose or switch to shortest length queue. There is no limit defined for number of Passengers in a queue or in a system.

BASIC QUEUING PROCESS

Passengers requiring service are generated over time by an input source. The required service is then performed for the Passengers by the service mechanism, after which the Passenger leaves the queuing system. We can have following two types of models: One model will be as Single-queue Multiple-Servers model (fig.1) and the second one is Multiple-Queues, Multiple-Servers model (fig.2) (Sheu, C., Babbar S. (Jun 1996)).

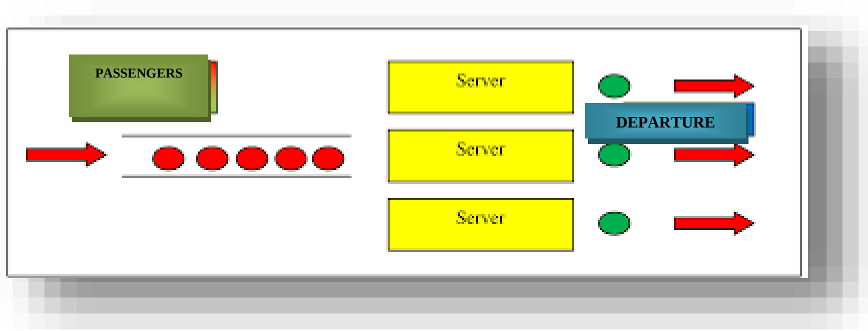


Fig. 1: Single Stage Queuing Model with Single Queue & Multiple Parallel Servers

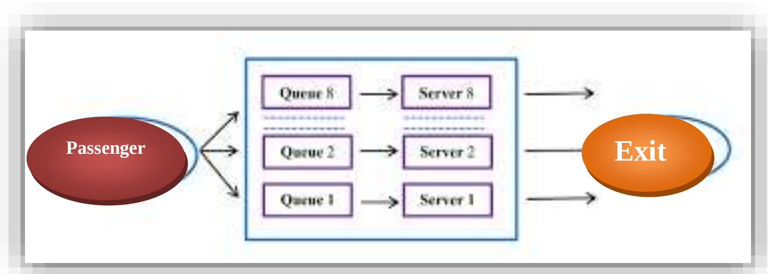


Fig. 2: Single Stage Queuing Model with Multiple Queue & Multiple Parallel Servers

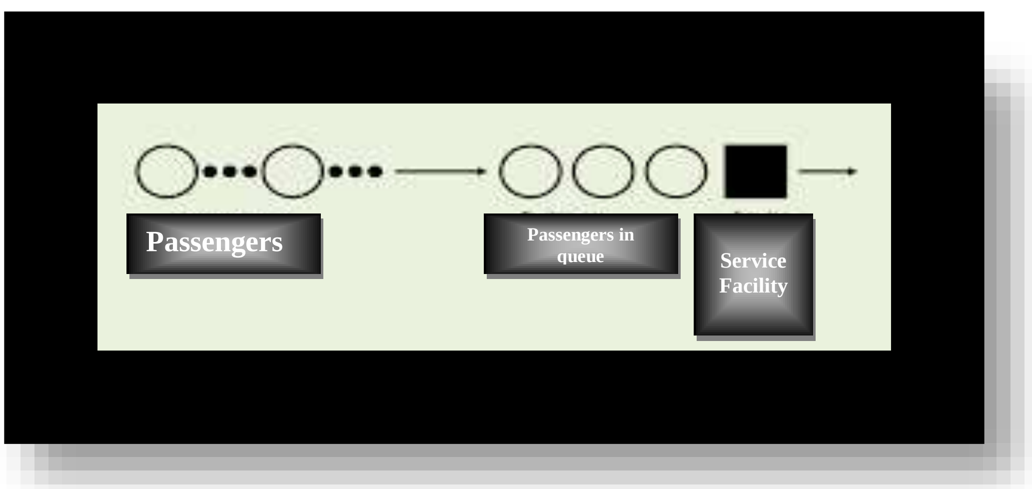


Fig. 3: Single Stage Queuing Model with Single Queue & Single Parallel Servers

In these models, three various sub-processes may be distinguished:

- **Arrival Process**: Includes number of Passengers arriving, several types of Passengers, deterministic or stochastic arrival distance, and arrival intensity. The process goes from event to event, i.e. the event "Passenger arrives" puts the Passenger in a queue, and at the same time schedules the event "next Passenger arrives" at some time in the future.

- **Waiting Process**: Includes length of queues, servers' discipline (First In First Out). This includes the event "start serving next Passenger from queue" which takes this Passenger from the queue into the server, and at the same time schedules the event "Passenger served" at some time in the future.
- **Server Process**: Includes a type of a server, serving rate and serving time. This includes the event "Passenger served" which prompts the next event "start serving next Passenger from queue".(Troitzsch, 2006)

The Queuing model is commonly labeled as M/M/c/K, where first M represents Markovian2 exponential distribution of inter-arrival times, second M represents Markovian exponential distribution of service times, c (a positive integer) represents the number of servers, and K is the specified number of Passenger in a queuing system. This general model contains only limited number of K Passenger in the system. However, if there are unlimited number of Passenger exist, which means K = Q, then our model will be labeled as M/M/c (Hillier & Lieberman, 2001.)

Parameters in Queuing Models (Multiple Servers, Multiple Queues Model)

- n - Number of total Passengers in the system (in queue plus in service).
- c - Number of parallel servers (Ticket window units in Railway Station).
- S - Arrival rate (1 / (average number of Passengers arriving in each queue in a system in one hour)).
- T - Serving rate (1 / (average number of Passengers being served at a server per hour)).
- cT - Serving rate when c > 1 in a system.
- r - System intensity or load, utilization factor (= l/(cμ)) (the expected factor of time the server is busy that is, service capability being utilized on the average arriving Passengers)

Departure and arrival rate are state dependent and are in steady-state (equilibrium between events) condition.

NOTATIONS & THEIR DESCRIPTION FOR SINGLE QUEUE AND PARALLEL MULTIPLE SERVERS MODEL (FIG.1) ASSUMING THE SYSTEM IS IN STEADY-STATE CONDITION

P_0: Steady − state Probabilities of all idle servers inthe system

$$\text{i.e. } P_0 = \left[\sum_{n=0}^{c-1} \frac{\gamma^n}{n!} + \frac{\gamma^c}{c!\,(1-\rho)}\right]^{-1} \text{ where } \gamma = \frac{\lambda}{\mu}$$

P_n: Steady − state Probabilities of n Passengers inthe system, i.e., $P_n = \frac{\lambda^n}{c!c^{n-c}\mu^n} P_0\ ; n > c$

L_q: Average number of Passengers inthe waiting line (queue) i. e., $L_q = \frac{\gamma^c}{c!(1-\rho)^2} P_0$

W_q: Average waiting time a Passenger spends waiting ibn the line excluding the service time

$$W_q = \frac{L_q}{\lambda}$$

There are no predefined formulas for networks of queues, i.e. multiple queues (fig.2). A complexity of the model is the main reason for that. Therefore, we use notations and formulas for single queue with parallel servers. In order to calculate estimates for multiple queues multiple servers' model, we may run simulation.

Expected length of each Queue

Besides service time, it is important to know the number of Passengers waiting in a queue to be served. It is possible that any Passenger would change his queue and choose another if find a shorter queue in another parallel server. In general, variability of inter-arrival and service time causes lines to fluctuate in length. Then question arises, what could be the estimated length of the queue in any server? These counts are a combination of input processes, that are: arrival point process, Poisson counting process (which counts only those units that arrive during the inter-arrival time and these units are conditionally independent on Poisson interval), and counting group of units being served within the Poisson interval. The above mentioned formula of Lq is defined for average queue length of the queuing system but does not evaluate a length of parallel queues.

We are next concerned about how to obtain solution for a queuing model with a network of queues? Such questions require running Queuing Simulation. Simulation can be used for more refined analysis to represent complex systems.

QUEUING SIMULATION

The queuing system is when classified as M/M/c with multiple queues where number of Passengers in the system and in a queue is infinite, the solution for such models are difficult to compute. When analytical computation of T is very difficult or almost impossible, a Monte Carlo simulation is appealed in order to get estimations. A standard Monte Carlo simulation algorithm fix a regenerative state and generate a sample of regenerative cycles, and then use this sample to construct a likelihood estimator of state. (Nasroallah, 2004) Although supermarket sales do not have regenerative situation but simulation here is used to generate estimated solutions.

Simulation is the replication of a real world process or system over time. Simulation involves the generation of artificial events or processes for the system and collects the observations to draw any inference about the real system. A discrete-event simulation simulates only events that change the state of a system. Monte Carlo simulation uses the mathematical models to generate random variables for the artificial events and collect observations. (Banks, 2001)

Discrete models deal with system whose behavior changes only at given instants. A typical example occurs in waiting lines where we are interested in estimating such measures as the average waiting time or the length of the waiting line. Such measures occur only when the Passenger enters or leaves the system. The instants at which changes in the system occurs identify the model's events, e.g. arrival and departure of the Passengers. The arrival events are separated by the 'inter-arrival time' (the interval between successive arrivals), and the departure events are specified by the service time in the facility. The fact that these events occur at discrete points is known as "Discrete-event Simulation." (Taha, 1997)

When the interval between successive arrivals is random then randomness arises in simulations. The time t between Passengers arrivals at Railway Station is represented by an exponential distribution; to generate the arrival times of the next Passengers from this distribution, we have $t = \frac{1}{\mu}\ln(1-R)$ where R = random number. (1 – R) is a compliment of R, so we can replace (1 – R) with R.

In keeping with the queuing theory models, simulated daily arrivals followed the Poisson probability law, and lengths of stay followed the negative exponential probability law (actually its discrete counterpart, the geometric probability law).

DISCRETE TIME ARRIVALS AND DEPARTURES

The column labeled discrete arrivals" uses the same assumptions as Young's model, except that there is a finite waiting line of non- emergent passengers. The column labeled "discrete and continuous" contains simulated values assuming (arbitrarily) that one-half of emergency arrivals occur between the normal (after- noon) admitting hour and the midnight census, 10 percent between the midnight census and the normal reservation hour, and the remaining percent occur after the reserved days have occurred but before the normal reservation hour. The average census in both simulations is the average 'm' right census, rather than the continuous average. In contrast to Young's results, the simulated values do not always exceed the theoretical values. Nor does the more realistic value of the "discrete and continuous" model always fall between the discrete model and the theoretical value. This difference is due to the finite backlog of passengers.

EARLY RESERVATION VERSUS TURN-AWAY

In the models of Young, Shonick, and Shonick and Jackson, all arriving emergency passengers who cannot be given a seat are turned away from the bogie being modeled. The frequency of "turn-away" is reduced in these models by specifying an occupancy level *B* above which only emergency arrivals are reserved. The alternative of early reservation was implemented in the simulation by scanning the list of reservation dates and reservation passengers with the fewest number of days remaining, until all emergency arrivals are considered. This is only done in the event that all seats, including the number designated for emergency reservations, are full.

Turning away emergency passengers lowers the reservation rate but does not affect the length of stay (passenger days per seat), whereas reservation early to consider emergencies changes the length-of-stay distribution and thereby lowers average length of route, but does not affect the reservation rate.

SEAT RESERVATIONS FOR NON EMERGENT PASSENGERS

The queuing theory models did not allow seats to be reserved one or more days in advance of reservation. However, in the discrete time simulation model, it is an easy matter to reserve seats one day in advance simply admits non emergent passengers before emergent passengers. An alternate policy is to allow any unfilled seat to be used if necessary for an emergency reservation, even if the scheduled passenger is due to arrive momentarily. In this case, the passenger who has been bumped is assumed to remain on the waiting list.

For the configuration, the magnitude of the improvement is slightly larger by eliminating reservations. Although the waiting list will increase if either action is taken, the effect on the waiting list appears to be different under the early reservation assumption than under the turn- away assumption.

But that is not the whole story. The effect of removing seat reservations is reversed when the waiting lines build up substantially. In the (26,4) configuration, there are not enough unrestricted seats to handle the non emergent load, so eliminating the reservation system exacerbates the problem. The waiting lists mount at an increasing rate and average census actually diminishes.

The conclusion is that “restricting reservations to emergencies only at high occupancy levels” and “eliminating the reservation of seats for non emergent passengers” have similar effects on the average census and average waiting lists (ignoring the very important behavioral implications of these policies). However, the exact magnitude of this effect is dependent on how emergencies are handled and the current arrival rate and waiting lists at the facility. The potential change in census is of both practical and statistical significance, but it must be carefully evaluated in each instance.

UNRESTRICTED BEDS NEEDED

We have noted that restricting non emergent reservations that effectively prohibits the reservation of a non emergent passenger per day, with the result that the waiting lines built up indefinitely. This effect is predictable in the queuing theory model. If it is generally valid, this inequality can provide a guide for setting the designated seat occupancy. As long as it is satisfied, we can be assured that all non emergent passengers can be admitted, eventually. However, we should do some sensitivity analysis here to model with overflows of emergency passengers turned away.

For our assumed seat unit with arrival rates of non emergent and emergent passengers per day and average length of stay, the minimum number of unrestricted seats that satisfies Shonick and Jackson’s inequality. (Some rather extensive number pushing is involved.) We have noted previously that the configuration was unable to handle the non emergent load under any of the simulation assumptions. Just as the preceding analysis predicts, the waiting line increases with no apparent limits.

However, the average waiting list is suspiciously large for the configuration with early reservation and no seat reservations for scheduled train. In fact, the waiting line at the *end* of the simulated days. When this simulation was continued, the trend continued.

The conclusion is that Shonick and Jackson’s inequality provides a guideline as to the necessary number of seats available for either emergent or non emergent reservation, but that systems that lose fewer passenger days than the Shonick and Jackson model predicts (either because of no seat reservations for non emergent passengers, or because of early reservation instead of turning away emergency overflows) must have more non restricted seats than the inequality specifies.

A SPECIAL CASE

If there are sufficient restricted seats to ensure that the facility is virtually never entirely full, then the method of handling overflows of emergencies becomes irrelevant. Moreover, no patient days are lost due to turn away or early reservations; thus, the long-run average census will be the ratio of the arrival rate to the service rate per server, the usual result in queuing models. In fact, Shonick and Jackson formulated an expression for the average census when *B* (seats available for either emergent or non emergent reservations) remained fixed as *S* (total seats) became infinite, but they did not show that the resulting equation[23] reduces to this simple ratio. The result is easily seen, and will not be included here.

A sequence of simulations was run with increasing seat complement *(S)* which provides for these two convergences in average census.

This sequence also points out one way in which the models investigated in this paper mimic the real world. As more seats are made available, more seats are filled (but not without limit, of course). For example, when the pressure of emergent passengers arriving to a full facility is relieved (seats are increased), fewer passengers are attempted early; thus, average length of stay increases and average census increases, even though demand (in terms of arrival rate) stays the same. Conversely, when the pressure increases (seats are decreased), length of stay drops. This would also occur if the increase in pressure were due to an increased arrival rate rather than a decrease in seats.

DISCUSSION AND CONCLUSIONS

The simulations reported here test the significance of some of the fundamental assumptions of the queuing theory approaches cited, and demonstrate that average census and likelihood of seat shortage vary significantly when assumptions of continuous arrivals/departures or turning away passengers are altered. The errors are serious enough that one should be cautious in applying the results in census planning whenever any significant probability exists that the facility will be full.

There are other assumptions, not tested here, that can be handled by simulation. Examples are variations in the arrival rate based on weekly patterns or annual patterns, and variations in the length of stay distribution, either by class or reservation (e.g., emergent versus non emergent), or by day of week. That these effects exist has been documented. However, Shonick chooses to sidestep this issue, saying, It is assumed that the planner would not wish to adopt as ‘desirable’ (seat) complements which are unnecessarily large because of avoidable day of the week census fluctuations resulting from administratively faulty reservation and get reservation policy. If one does not assume that this pattern should be ignored, then the use of steady arrival rates is called into question. Analysis of the impact of this assumption on average census and seat shortage will be of great interest.

The implementation of models intended for long-range seat planning is hindered by data requirements that are difficult to satisfy. For example, estimating the length of stay distribution is not straightforward in an institution that is under pressure due to lack of seats. In order to accurately represent the situation faced by the railway, the model needs an input consisting of what the length of stay and arrival distributions there were no early reserved seats or passengers turned away due to seat pressures, and 2) utilization of seats was always appropriate. This information is extremely difficult to obtain. Again, in order to accurately represent the results of differing admissions policies, the model must have the pattern of demands according to when the request for reservation was made, rather than when the passenger get reservation. Also, if a passenger is purposely demanded to the wrong seat because of census pressure, he should be counted as a demand for the appropriate seat (rather than the one to which he was reserved), so that the model will place him where he belongs if more seats are made available.

These difficulties are pointed out to demonstrate that a highly refined model may not be appropriate for long-range seat planning since the errors inherent in forecasting the input to the model are likely to be substantial. In the final analysis, it may be decided that a model for long-range seat planning need not represent every detail of railway operations. It is hoped that continued analysis will bring to light the most important details, and that a generally useful model with a moderate amount of detail will result.

We have carefully studied the results of two mathematical modeling efforts and found them both to be much more restrictive in their usefulness in railway seat planning than may be suspected at first reading. Those papers were not exceptional in that regard, however. It is hoped that this paper will promote the recognition that sensitivity analysis should be part of all operations research, and is particularly important for models that are being suggested for general use.

REFERENCES

1. Abolnikov, L., Dshalalow, J. E., Dukhovny, A. M. (1990), "On Some Queue Length Controlled Stochastic Processes, "Journal of Applied Mathematics and Stochastic Analysis, Vol. 3, No. 4.
2. Adan, I.J.B.F., Boxma1, O.J., Resing, J.A.C. (2000), "Queuing models with multiple waiting lines," Department of Mathematics and Computer Science, Eindhoven University of Technology, Banks, J., Carson, J. S., Nelson, B. L., Nicol, D. M. (2001), Discrete-Event System Simulation, Prentice Hall international series, 3rd edition, p24–37.
3. Bhatti, S. A., Bhatti, N. A. (1998), Operations Research – an Introduction, Department of ComputerScience, Quad-e-Azam University, p.315–356.
4. Hillier, F. S., Lieberman, G. J. (2001), Introduction to Operations Research, McGraw-Hill higher education,7th edition, p834–8.
5. Jensen, Paul A. (2004), "Queuing models," Operations Research Models and Methods,www.me.utexas.edu/~jensen/ORMM/models/unit/queue/index.html.
6. Keith G. Calkins (May 2005), "Queuing theory and Poisson distribution," Statistical Probabilities andDistributions, Ch. 10, www.Andrews.edu/~calkins/math/webtexts/prod10.html.
7. Nasroallah, A. (2004), "Monte Carlo Simulation of Markov Chain Steady-state Distribution," Extract Mathematics, Vol. 19, No. 2, p279-288.
8. Patel, J. J., Chadhaury, R. M., Patel, J. M., (2019), "Queuing theory and its application at railway ticket window", Journal of information, knowledge and research in information technology, Vol.02, Issue 01, p 99-113.
9. Sheu, C., Babbar S. (Jun 1996), "A managerial assessment of the waiting-time performance foralternative service process designs," Omega, Int. J. Mgmt Sci. Vol. 24, No. 6, pp. 689-703.
10. Taha, Hamdy A. (1997), Operations Research an Introduction, PHIPE Prentice, 6th edition, p607–643.
11. Tsuei, Thin-Fong; Yamamoto, W., "A Processing queuing simulation model for multiprocessorsystem performance analysis," Sun Microsystems, Inc.
12. Troitzsch, Klaus G., Gilbert, Nigel (Sep 2006), "Queuing Models and Discrete Event Simulation,"ZUMA Simulation Workshop 2006.

13. J.K.Sharma "Operation Research Theory and Application"Third edition, Chepter-16, Page 712-781.
14. Operations Management – Focusing on Quality & competitiveness, EW University, Ch.16 education, 6th edition, p834–8