Geometric Alignment Effect of Applied Magnetic Fields on Nonparaxial Laser Ray self-focusing via Relativistic Plasma

Abdulkareem Thjeel Jabbar [1], Munther B. Hassan [2]

[1] Directorate of Education Al-Muthanna, Ministry of Education, Republic of Iraq

[2] Department of Physics, Faculty of Science, University of Kufa, Najaf, Iraq

Abstract

This article investigates the geometric arrangement influence of external magnetic fields (Parallel and transverse magnetic fields with respect to laser beam propagation) on the laser rays propagation inside relativistic plasma. Adopting the nonparaxial theory of laser rays propagation, the results have showed that the increase of magnetic fields in both parallel and transverse regimes are leading to high self-focusing of laser beam. The parallel magnetic field alignment have illustrated a higher self-focusing, leading to higher intensity of laser beam inside plasma, comparing with transversal magnetic field.

Keywords: Relativistic Nonlinearity, Self-Focusing, Nonparaxial Laser Ray, Cyclotron Frequency.

1. Introduction

Nowadays the relativistic nonlinear interactions between laser and plasma have widely enticed researchers in many theoretical and practical fields [1-4]. There are different applications [5-13] that requires a very high intense laser in order of 10^{18} W/cm^2. In this regards, in order to reach such a level of laser intensity, the focusing of laser beam via plasma is the solution as long as there is no probability of plasmas breakdown. Munther B. Hassan et. al. [14] have introduced a theoretical study of ponderomotive nonlinear interaction between high power laser beams with magnetized plasma. They have proved that the geometric of applied magnetic fields for both parallel and transversal cases leading to change the self-focusing in paraxial region. .For larger realistic and applicable aims, this study has been extended to include the laser beam propagation in the nonparaxial region.

In sections (2) and (3), the appropriate expressions of the nonparaxial laser beam self- focusing via magnetized plasma have been derived for both parallel and transverse magnetic fields respectively. To more understand of the self-focusing phenomenon, so deep discussion of numerical results and the most distinguished conclusions have been introduced in section (4) and section (5) respectively.

2. Self-focusing of nonparaxial laser beam in parallel magnetic field

Postulating a Gaussian laser beam is propagating along an external magnetic field B_0 in z-direction. Hence the electric field($\vec{E}_{0+}$) of laser ray may be given as

$$\vec{E}_{0+} = \vec{A}_{0+} \exp i(\omega_0 t - k_{0+} z) \qquad (1)$$

Where $\vec{A}_{0+} = \vec{E}_x + i\vec{E}_y$ is the electric field amplitude, ω_o is the angular frequency and k_{0+} is the wave vector.

To govern the laser beam propagation via magnetized plasma, one may introduce the wave equation as following:

$$\nabla^2 \vec{E}_{0+} - \vec{\nabla}\left(\vec{\nabla} \cdot \vec{E}_{0+}\right) + \frac{c^2}{\omega_0^2} \varepsilon_+ \vec{E}_{0+} = 0 \qquad (2)$$

The relativistic dielectric tensor ε_+ may be evaluated according upon Munther *et. al.* technique to become [15]

$$\varepsilon_+ = \varepsilon_{0+} + \varepsilon_{2+} \vec{A}_{0+} \vec{A}_{0+}^* = 1 - \frac{\left(\frac{\omega pe}{\omega_0}\right)^2}{\left(1 - \frac{\omega ce}{\omega_0}\right)} + \frac{\left(\frac{\omega pe}{\omega_0}\right)^2}{\left(1 - \frac{\omega ce}{\omega_0}\right)} a_{r+} \vec{A}_{0+} \vec{A}_{0+}^* \ (3)$$

It is worth noting that the dielectric tensor consists of a linear $\left(\varepsilon_{0+} = 1 - \frac{\left(\frac{\omega pe}{\omega_0}\right)^2}{\left(1 - \frac{\omega ce}{\omega_0}\right)}\right)$ and a nonlinear $\left(\varepsilon_{2+} \vec{A}_{0+} \vec{A}_{0+}^* = \frac{\left(\frac{\omega pe}{\omega_0}\right)^2}{\left(1 - \frac{\omega ce}{\omega_0}\right)} a_{r+} \vec{A}_{0+} \vec{A}_{0+}^*\right)$ parts.

Where $\alpha_{r+} = \frac{e^2}{2m_0^2 c^2 \omega_0^2} \cdot \frac{1}{(1 - \frac{\omega_{ce}}{\omega_0})^2}$ is the relativistic nonlinearity

Factor in presence of parallel magnetic field.

Employing Eq. (3), the wave equation (Eq. (2)) may be rewritten as follows:

$$\frac{\partial^2 A_{0+}}{\partial z^2}+\frac{1}{2}\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{0zz}}\right)\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)A_{0+}+\frac{\omega_0^2}{c^2}(\varepsilon_{0+}+\varepsilon_{2+}A_{0+}A_{0+}^*)A_{0+}=0 \qquad (4)$$

The product of non-linear part and $\frac{\partial^2 A_{0+}}{\partial x^2}$ or $\frac{\partial^2 A_{0+}}{\partial y^2}$ has been neglected [16]. To solve eq. (4) assuming $A_{0+}=A'_{0+}\exp i(\omega_0 t-k_{0+}z)$ where $A_{0+}=A'_{0+}$ is complex amplitude, we obtain:

$$-2ik_{0+}\frac{\partial A'_{0+}}{\partial z}+\frac{1}{2}\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{0zz}}\right)\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)A'_{0+}+\frac{\omega_0^2}{c^2}(\varepsilon_{2+}A'_{0+}A'^*_{0+})A_{0+}=0 \qquad (5)$$

Suggesting a 2-dimensional Gaussian beam ($\frac{\partial}{\partial y}=0$) with introducing an phase $A`_{0+}=A_{0+}^0\exp(i\,k_{0+}S_+)$, where ($A_{0+}^0$) is the real functions and (S_+) is the phase for the laser beam inside magnetic, so eq. (5) after segregation real and imaginary parts, can be written as following [17] :

$$2\frac{\partial S_+}{\partial z}+\frac{1}{2}\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{0zz}}\right)\left(\frac{\partial S_+}{\partial z}\right)^2+\frac{1}{2\,k_{0+}^2\,A_{0+}^0}\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{0zz}}\right)\frac{\partial^2 A_{0+}^0}{\partial x^2}=\frac{\varepsilon_{2+}}{\varepsilon_{0+}}(A_{0+}^0)^2 \quad (6)$$

$$\frac{\partial (A_{0+}^0)^2}{\partial z}+\frac{1}{2}(A_{0+}^0)^2\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{0zz}}\right)\frac{\partial^2 S_+}{\partial x^2}+\frac{1}{2}\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{0zz}}\right)\frac{\partial S_+}{\partial x}\frac{\partial (A_{0+}^0)^2}{\partial x}=0 \qquad (7)$$

In the non-paraxial ray theory, S_+ can be expanded to $S_+=\frac{1}{2}x^2\beta_+(z)+\varphi_+(z)(\varphi_+$ is a constant which does not depend on$x$). Introducing initially Gaussian beam amplitude [18],

$$A_{0+}=\frac{E_{00}}{f_{0+}}\left(1+\alpha_{00}\frac{x^2}{x_0^2 f_{0+}^2}+\alpha_{02}\frac{x^4}{x_0^4 f_{0+}^4}\right)^{1/2}e^{-\frac{x^2}{2x_0^2 f_{0+}^2}} \qquad (8)$$

Where α_{00} and α_{02}represent are the non-paraxial coefficients.

$$S_+=\left[\frac{1}{\left(1+\frac{\epsilon_{o+}}{\epsilon_{zz}}\right)}\frac{1}{f_{0+}}\frac{df_{0+}}{dz}\right]x^2+\left[\frac{S_{2+}}{x_0^4}\right]x^4 \qquad (9)$$

$$\beta_+(z)=2\left(1+\frac{\epsilon_{0+}}{\epsilon_{zz}}\right)^{-1}\frac{1}{f_{0+}}\frac{df_{0+}}{dz} \qquad (10)$$

The final equations of laser beam self-focusing in non-paraxial region can be written as:

$$\frac{d^2 f_{0+}}{dz^2}=\frac{\left(1+\frac{\epsilon_{0+}}{\epsilon_{zz}}\right)^2}{4k_{0+}^2\,x_0^4}\frac{1}{f_{0+}^3}(8\alpha_{02}-3\alpha_{00}^2-2\alpha_{00}+1)-\frac{\left(1+\frac{\epsilon_{0+}}{\epsilon_{zz}}\right)}{2\epsilon_{0+}} \qquad (11)$$

$$\frac{\partial S_+}{\partial z}=\left[\frac{\epsilon_{r+}(1-\alpha_{00}+\alpha_{02})\,{E_{00}}^2}{2\epsilon_{0+}{f_{0+}}^6}\right]-\left[\frac{\left(1+\frac{\epsilon_{0+}}{\epsilon o_{zz}}\right)}{2{x_0}^2{k_{0+}}^2{f_{0+}}^6}\right]({\alpha_{00}}^2-7\alpha_{00}\alpha_{02}+{\alpha_{00}}^3-2\alpha_{02})-\frac{4S_{2+}}{f_{0+}}\frac{df_{0+}}{dz} \qquad (12)$$

$$\frac{\partial \alpha_{00}}{\partial z} = \frac{-8 f_{0+}{}^{2} S_{2+}}{x_o{}^2}\left(1+\frac{\epsilon_{o+}}{\epsilon o_{zz}}\right) \qquad (13)$$

$$\frac{\partial \alpha_{02}}{\partial z} = \left[\frac{4 f_{0+}{}^{2} S_{2+}}{x_0{}^2} - \frac{12\alpha_{00} f_{0+}{}^{2} S_{2+}}{x_0{}^2}\right]\left(1+\frac{\epsilon_{0+}}{\epsilon_{0zz}}\right) \qquad (14)$$

The first term in the right hand side of the Eq.11 is referring to the natural diffraction effect while the second term of it is representing the nonlinear self-focusing effect. It is more importance to mention that the oscillation pattern of the laser wave inside plasma is appearing as a result of the rivalry between these two effects (i.e. diffraction effect and self-focusing).

3. Self-focusing of nonparaxial laser beam in transverse magnetic field

In presence of the transversal magnetic field along z- axis, the electric field of the laser wave along the x- axis may be written as:

$$\vec{E} = \vec{A} \exp i(\omega_0 t - k_0 x) \qquad (15)$$

Where the laser wave amplitude $\vec{A} = \hat{x} A_x + \hat{y}\, A_y$ is complex function of space which is written as:

$$\vec{A} = A_o \exp i(\, k_o S\ ) \qquad (16)$$

A_o and S are representing the real and the phase functions of the laser ray amplitude.

The wave equation deciding the laser ray propagation through plasma is written as

$$\nabla^2 \vec{E} = \vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \frac{\omega_o{}^2}{c^2}(\varepsilon_r . E) \qquad (17)$$

The dielectric tensorε_r of magnetized plasma is responsible the nonlinear behavior of laser wave. Motivating the relativistic nonlinearity and following Monika et. al.[19], one may introduce as

$$\varepsilon_r = \varepsilon_0 + \varepsilon_2 E_y E_y^* = 1 - \frac{\frac{\omega_{pe}^2}{\omega_0^2}\left(1-\frac{\omega_{pe}^2}{\omega_0^2}\right)}{\left(1-\frac{\omega_u^2}{\omega_0^2}\right)} + \left[\frac{\frac{\omega_{pe}^2}{\omega_0^2}\left(\left(1-\frac{\omega_{pe}^2}{\omega_0^2}\right)^2+\frac{\omega_{ce}^2}{\omega_0^2}\right)}{\left(1-\frac{\omega_u^2}{\omega_0^2}\right)^2}\right]\alpha_r E_y E_y^*, \quad (18)$$

This equation contains a linear nonrelativistic term $\varepsilon_0 = 1 - \frac{\frac{\omega_{pe}^2}{\omega_0^2}\left(1-\frac{\omega_{pe}^2}{\omega_0^2}\right)}{\left(1-\frac{\omega_u^2}{\omega_0^2}\right)}$ and nonlinear relativistic term $\varepsilon_2 E_y E_y^* = \left[\frac{\frac{\omega_{pe}^2}{\omega_0^2}\left(\left(1-\frac{\omega_{pe}^2}{\omega_0^2}\right)^2+\frac{\omega_{ce}^2}{\omega_0^2}\right)}{\left(1-\frac{\omega_u^2}{\omega_0^2}\right)^2}\right]\alpha_r E_y E_y^*$.

Where $\alpha_r = \frac{1}{4}\left(\frac{e}{m_0\omega_0 c}\right)^2\left[1+3\left(\frac{\omega_{ce}}{\omega_0}\right)^2+4\left(\frac{\omega_{ce}}{\omega_0}\right)^2\left(\frac{\omega_{pe}}{\omega_0}\right)^2\right]$ is the relativistic nonlinearity factor in presence of transverse magnetic field.

As in section 2 and taken in our account the influence of transverse magnetic field on laser beam propagation via plasma, the final equations related with the self-focusing in nonparaxial region will become

$$\frac{\partial f^2{}_o}{\partial x^2} = \left[\frac{1}{k_o{}^2 x_o{}^4} \quad \frac{1}{f_o{}^3}\right](1\text{-}2\alpha_{oo} - 3\alpha_{oo}{}^2 + 8\alpha_{o2})\text{-}(1\text{-}\alpha_o)\left[\frac{\epsilon_r E_{oo}{}^2}{\epsilon_0 x_o{}^2}\frac{1}{f_o{}^2}\right] \quad (19)$$

$$\frac{\partial S}{\partial x} = \left[\frac{\epsilon_r E_{oo}{}^2}{2\epsilon_o f_o{}^6}\right](1-\alpha_{00}+\alpha_{02}) - (\alpha_{00}{}^2 - 7\alpha_{00}\alpha_{02} + \alpha_{00}{}^3 - 2\alpha_{02})\left[\frac{\left(1+\frac{\epsilon_0}{\epsilon o_{zz}}\right)}{2x_o{}^2 k_o{}^2 f_o{}^6}\right] - \frac{4S_2}{f_o}\frac{df_o}{dx} \quad (20)$$

$$\frac{\partial \alpha_{00}}{\partial x} = \frac{-8f_o{}^2 S_2}{x_o{}^2}\left(1+\frac{\epsilon_o}{\epsilon_{oxx}}\right) \quad (21)$$

$$\frac{\partial \alpha_{02}}{\partial x} = \left[\frac{4f_o{}^2 S_2}{x_o{}^2} - \frac{12\alpha_o f_o{}^2 S_2}{x_o{}^2}\right]\left(1+\frac{\epsilon_0}{\epsilon_{0xx}}\right) \quad (22)$$

Both set of the eqs. (11-14) and (19-22) for parallel and transverse magnetic field respectively have been numerically solved to understand the geometric arrangement influence on laser beam self-focusing inside plasma.

3. The numerical results and discussion

In this study, the high intense (CO) pulsed laser is coupling with magnetized plasma in the nonparaxial region. The typical parameters of the laser plasma interaction are: The angular frequency of laser ray $\left(\omega_0 = 3.77\times10^{14}\, rad.\sec^{-1}\right)$ identical to the laser frequency $\left(\nu = 6\times10^{13}\sec^{-1}\right)$, the initial laser ray intensity $\left(I = 10^{17} W/cm^2\right)$, the initial laser ray radius $\left(x_0 = 40\,\mu\text{m}\right)$, the plasma densities $\left(n_e = 2.8\times10^{18} cm^{-3}\right)$ identical to the plasma frequency $\left(\omega_{pe} = 0.9\times10^{14}\, rad.\sec^{-1}\right)$, the applied magnetic fields $\left(B = \left(0.2, 0.6, 1\right) MG\right)$ identical to the cyclotron frequencies $\left(\omega_{ce} = (0.37, 1.13, 1.88)\times10^{13}\, rad.\sec^{-1}\right)$, the non-paraxial coefficients $\left(\alpha_{00} = \alpha_{02} = 0.01\right)$.

In presence of a parallel magnetic field, figure (1) is demonstrating the vibrating pattern of laser ray during its travelling inside plasma due to the competition between the linear and non-linear terms (see Eq. 11). It is worth to note that, along the normalized propagation distance, the decreasing of laser beam width parameter is corresponding to the increasing of magnetic field magnitudes. In general same behaviors for transverse magnetic field have been recorded but in weaker modality (see figure 2). To investigate the main behavior difference in both parallel and transverse magnetic fields, figure (3) explains that the laser beam self-focusing in parallel magnetic field case is greater comparing with transversal magnetic field state.

6. Conclusions

One may see that the applied magnetic fields have a considerable role in enhancement the self-focusing phenomenon in both parallel and transversal states. In a parallel state, this role will be more evident on the self-focusing laser ray comparing with transversal magnetic field state. Whenever the power of the incident laser beam is greater than the critical power, the laser wave will demonstrate an oscillating behavior as a result of the vying between the nature diffraction phenomenon (diverging the laser beam) and self-focusing term (converging the laser beam).

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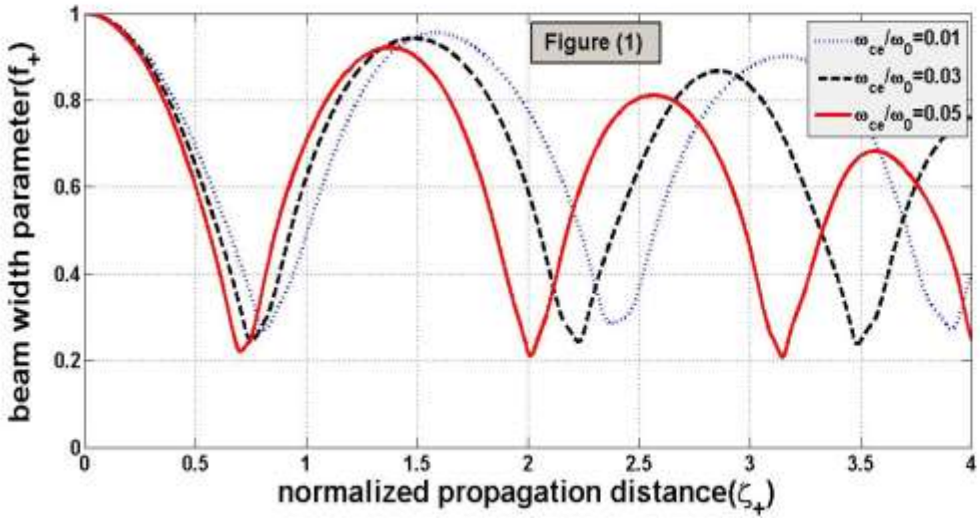

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Fig.1. (Color online) Variation of beam width parameter (f_+) in nonparaxial region along the normalized propagation distance $\left(\zeta_+ = \frac{z}{k_{0+}x_0^2}\right)$ in presence of parallel magnetic field where dotted blue line, black dashed line and sold red line represent $(\omega_{ce} = 0.01\omega_0$, $\omega_{ce} = 0.03\omega_0$ and $\omega_{ce} = 0.05\omega_0)$ respectively.

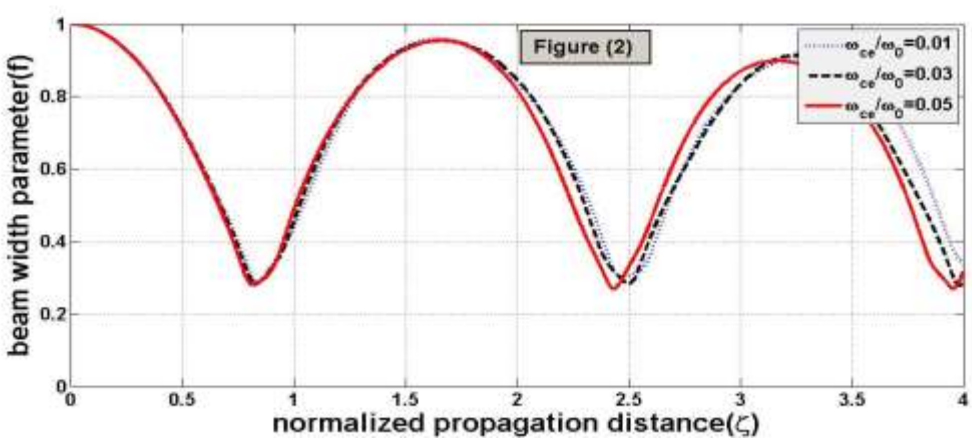

Fig.2. (Color online) Variation of beam width parameter (f) in nonparaxial region along the normalized propagation distance $\left(\zeta = \frac{z}{k_0 y_0^2}\right)$ in presence of transverse magnetic field where dotted blue line, black dashed line and sold red line represent $(\omega_{ce} = 0.01\omega_0$, $\omega_{ce} = 0.03\omega_0$ and $\omega_{ce} = 0.05\omega_0)$ respectively.

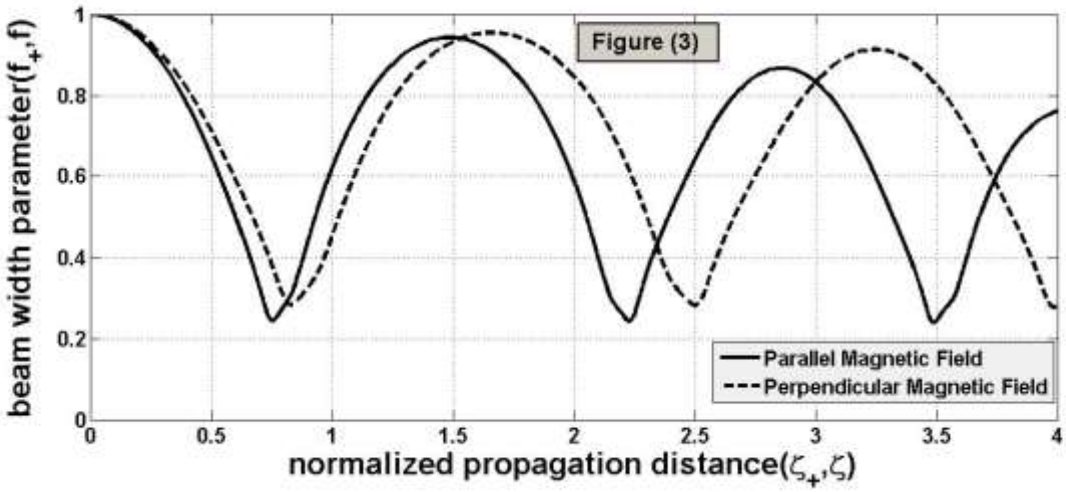

Fig.3. (Color online) Variation of beam width parameter (f_+, f) in nonparaxial region along the normalized propagation distance (ζ_+, ζ) in presence of parallel and transverse magnetic fields

References

[1] D.N.Guptak, H. Suk, K.Ayinash, (2013), Transient self-focusing of an intense laser pulse in magnetized plasmas under non-paraxial approximation. Laser Part Beams; 31(02), pp. 307-312.

[2] H. Kumar, M. Aggarwal, Richa, T.S. Gill, (2018), Self-focusing of an elliptic-Gaussian laser beam in relativistic ponderomotive plasma using a ramp density profile, J. Opt. Soc. Am. B: Opt. Phys ; 35 (7), pp.1635-1641 ,https://doi.org/10.1364/JOSAB.35.001635 .

[3] S.D. Patil, M.V. Takale, T.S. Gill, (2015), Effect of light absorption on relativistic self-focusing of Gaussian laser beam in plasma, Eur. Phys. J. D ;69 (163),pp.1-4 .

[4] H.Kumar, M. Aggarwal, (2018), Self-focusing an elliptic-Gaussian laser beam in relativistic ponderomotive plasma using a ramp density profile, J Opt Soc Am B; 35(7), pp.1635–1641.

[5] R. Kaur, T.S. Gill, R. Mahajan, (2017), Relativistic effects on evolution of a q-Gaussian laser beam in magnetoplasma: application of higher order corrections, Phys. Plasmas; 24(5), 053105, https://doi.org/10.1063/1.4983309,pp.1-7 .

[6] K.Sharma, (1978), Transverse self-focusing and filamentation of a laser beam in a magnetoplasma, J. Appl. Phys.; 49 (4), pp. 2396-2400.

[7] A. Dobpoiu, C. Otani , K. Kawase, (2006), Terahertz-wave sources and imaging applications, Meas. Sci. Technol. ;17 (11),pp.R161-R174 .

[8] H.Sun ,P. Huang ,Y. Tzeng , J.Huang ,C.D. Lin ,C. Jin ,M. Chen ,(2017), Extended phase matching of high harmonic generation by plasma-induced defocusing. Optica; 4(8), pp.976-981.

[9] RA. Ganeev, (2007), High-order harmonic generation in a laser plasma: a review of recent achievements. J Phys B: Atom, Mol Opt Phys.; 4o (22), PP. R213-R253.

[10] P Zhang, AGR.Thomas, (2015), Enhancement of high-order harmonic generation in intense laser interactions with solid density plasma by multiple reflections and harmonic amplification. Appl Phys Lett.; 106: (13), pp. 1-4 .

[11] F.Jahangiri, M. Hashida, S. Tokita, T. Nagashima, M. Hangyo, S. Sakabe, (2013), Enhancing the energy of terahertz radiation from plasma produced by intense femtosecond laser pulses. App. Phys Lett; 102(19), pp. 1-4.

[12] P. Malik, SC. Sharma, R. Sharma, (2018), Coherent terahertz radiation from beam-driven upper hybrid wave in magnetostatic plasma. J Electromag Waves Appl; 32 (17), pp.2195-2209.

[13] A. Munafò et al., (2020), A computational model for nanosecond pulse laser-plasma interactions, J. Comput. Phys., 406, doi: https://doi.org/10.1016/j.jcp.2019.109190 pp. (1-38).

[14] M.B.Hassan,A. Hassan , A.S.Obeis,Static magnetic field influence on pondromotive self-focusing of laser beam through plasma, Natural and Social Sciences ; 4(5): , May (P): 2347-4580 9-20 © Impact Journals.

[15] M. B. Hassan, A. H. Aljanabi, R. P. Sharma, M. Singh and, J., (2012), Plasma Physics, Terahertz generation by the high intense laser beam, Journal of Plasma Physics, Cambridge University;78(5): ,pp.553-558 .

[16] S. A. Akhmanov, A. P. Sukhnorukov and R. V. Khokhlov, (1968), self-focusing and diffraction of light in a nonlinear medium, Sov. Phys. Usp; 10(5), pp. 609-636.

[17] M. S. Sodha,. , A. K. Ghatak, V. K. Tripathi, (1974a.), Self- Focusing of Laser Beam in Dielectric, Plasma and Semiconductors,Tata-McGraw-Hill, Delhi.

[18] H. A. Salih, R. P. Sharma, M. Rafat, (2004), Plasma wave and second-harmonic generation of intense laser beams due to relativistic effects. Phys. Plasmas; 11(6), pp.3186-3190.

[19] M. Singha , M. B. Hassan , A. H. Al- Janabi, .R. P. Sharma, N. Kumar,(2016), Study of Terahertz Generation in Magnetized Plasma Via Self Focused Ultra-Relativistic Laser Beam. IOSR Journal of Applied Physics; 8(5) ,pp. 37-45 .