Quadruple moments (Q_o) and root mean square radius $<r^2>^{1/2}$ of even –even isotopes of ($_{58}$Ce) and ($_{60}$Nd)

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Abstract :

In this work Some forms of double- cores nuclear marital have been studied. The investigation has been done by deformation indications through the following two methods: Method number 1: Four-pole nuclear deformation (β_2) depending on the potential to transfer (*B) (E2;0$^+\rightarrow$2$^+$)*↑. Method number 2: Four-pole nuclear deformation indications (δ) depending on self-determination quad pole (Q_o), in order to get deformation indications values of quad (β_2) pole ,the possibility of four-electrode *B* (*E2*;$0^+\rightarrow2^+$) was calculated by using the better equation universally (Global Best fit). While deformation indications quad pole (δ) which have been calculated by applying the relationship between (δ) and self-determined four-pole (Q_o) and the square root of the average radius ($<r^2>^{1/2}$). By drawing deformations indications (2β, δ) as a function of the number of neutrons (N), the relationship between these two indications has been studied. Through this relationship, it has clearly been noted that the deformation of the nucleus decreases every time that the number of nucleus is closer to magic numbers which means the spherical nuclei are in the magic numbers. In order to achieve the main goal (identification forms of nuclei through deformation indications) and reach to a decision whether the studied nuclei had oval or spherically form through calculating the small elliptical axis a (minor) and large (b).

Keyword (Global Best fit, minor, neutrons, nuclei, β_2, δ, Q_o)

Introduction : The nucleus interaction with an external electromagnetic field caused a transmission [1]. When gamma-rays are released between two agitated states in a nucleus, the bounced momentum could be about (1 part of 105) , so it can be ignored. The variance in the nuclear states energies that is involved in the transmission, is generally between (0.1 to 10 MeV) [2]. For gamma-ray energy, there will be a difference in angular momentum for the emitted photons. The disintegration possibility is the total emission of the partial prospects of these different sorts of gamma-rays [2].

Description of the calculation:

A direct information is given by the limitation of disintegration possibilities of nuclear states on the structure of the initial and final states and this can be exposed in highly collective and distorted structures within the nucleus [3]. In the electromagnetic transmission, valence and total angular momentum are preserved. If the flirtation and valence of the initial state is(J_i^{π})and that of the final state is (J_f^{π}) , a simple base on the photon multipolarity is given by the keeping of the total angular momentum L [4] :

$$|J_i - J_f| \leq L \leq J_i + J_f \quad \text{……….. (1)}$$

Photons can have only integration values of L; the value (L = 0) is planned out as a result of the fact that electromagnetic waves are transversed in nature, when (L = 1), the radiation was theoretically shown as correspondence the one to that was emitted by a vibrating dipole and for L=2 to the one from a quadruple, et cetra [2]. There are two classes of radiations for both (L) values, electric and magnetic multipole radiations, which have a difference in their valence. When (L) is even, the electric multipole has even parity, and when (L) is odd, parity is odd too . When (L) is even, the magnetic multipole radiation has odd parity, and when(L) is odd, even parity is odd too[5].

Quadruple Deformation Parameter β_2 : This parameter can measure the protraction of the axially proportional shape, presuming that a nuclide with uniform charge is distributed out to the distance $R(\theta, \emptyset)$ (the two polar angle) and zero charge beyond, (β_2) is associated with B (E2) as below [6].

$$\beta_2 = \frac{4\pi}{3ZRo^2}[B(E2)\uparrow\frac{e^2\,b^2}{e^2}]^{1/2} \quad \text{……………..(2)}$$

Where: the average radius nuclear is (R_o) that can be acquired from this equation:

$$R_O{}^2\text{=0.0144 } A^{2/3}\, barn \quad \text{……………..(3)}$$

Mean Square Charge Radius$< r^2 >$: This can be defined as the charge deployment that is changed from one isotope to another. The Mean Square Charge Radius$< r^2 >$ is traditionally called “nuclear volume shift” or “field shift”. This effectiveness can't be straightly determined, because there is no existence for the point-charge nucleus. Information about radius differences can be gained from the isotope shifts measurement. The important tool for looking at the nuclear systems behavior is the radius sensitivity to give details of the nuclear structure such as shell effects and deformation. A good approximation of the nuclear quantity that controls the isotope shift is the second radial moment of the nuclear charge apportionment [7].

The quantity($< r^2 >$), it is also called "nuclear mean square charge radius". Isotope shifts have information about the quantity changing which is considered as a function of the neutron number,($< r^2 >$) mean squared charge distribution radius that can be calculated by the following equations[8].

$$< r^2 >= 0.63\left[1.2A^{1/3}\right]^2 \qquad A\leq 100 \qquad \ldots\ldots\ldots\ldots (4)$$

$$< r^2 >= \frac{\left[0.63\, R_o^2\, (1+\frac{10}{3}(\frac{\pi\, a_o}{R_o})^2\right]}{\left[1+(\frac{\pi a_o}{R_o})^2\right]} \qquad A>100 \qquad \ldots\ldots\ldots\ldots.(5)$$

Where: (R_o) Radial Woods Saxon Potential Parameters that can be gained as :

$$R_o=1.07A^{1/3} fm \qquad \ldots\ldots\ldots.(6)$$

Where: (a_o) is acquired from the fast electrons scattering data ($a_o=0.55\ fm$) [8].

Intrinsic Quadrupole Moment Q_o: General electric charges allocation can be clarified by its quadruple moments, net charge and higher order moments [9]. It can also be said that the electric quadrupole moment can calculate the diffraction of the charge distribution from spherical symmetry. As a result of this , its measurement can provide information about the shape and nucleus deformation. The intrinsic electric quadrupole moment (Q_o) is measured in the intrinsic body-fixed frame [10]. In order to study the spatially extended particle shape is to determine its intrinsic quadrupole momentQ_o[10]:

$$Q_o = \int d^3\, r\rho(r)(3z^2 - r^2) \quad \ldots\ldots\ldots\ldots(9)$$

Where: $\rho(r)$ is the proton radial charge density and (r) is the charge radius.

When the charge density is concentrated straight on the z-orientation (symmetry axis of the particle), the term symmetrical to ($3z^2$) controls, Q_o is positive , and the particle is prolate. And in opposite, when the charge density is concentrated in the tropical plane vertical to z, the term symmetrical to (r^2) takes over, Q_o is negative and the particle is oblate[10]. For an axially symmetric nucleus, the (Q_o) is associated with the reduced electric quadrupole transsmision possibality B(E2) between the (0+) ground state and the first-excited (2+) status [6]:

$$Q_o = \left[\left(\frac{16\,\pi}{5}\right)\frac{B(E2)e^2b^2}{e^2}\right]^{1/2} \qquad \ldots\ldots\ldots\ldots.(10)$$

From this equation , the intrinsic quadrupole moment Q_o unit of barn (b) can be shown.

Quadrupole Deformation Parameters δ : In the conventional electrodynamics ,in order to get a simple model for a non-spherical homogeneous charge distribution is a rotatory ellipsoid with charge (z), major axis(b), and minor axis (a) as shown in fig.(1) [10] :

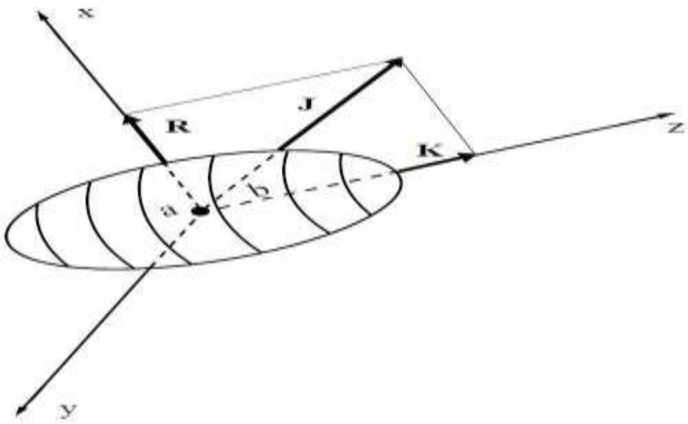

Fig.(1):The shape of nuclei when its prolate shape is with intrinsic spin (K) [10].

As the degree of nucleus shape deviation from nucleus spherical is measured by the quadrupole deformation parameter (δ) [6] , the quadrupole deformation parameter (δ) can be calculated from this equation[11]:

$$\delta = \frac{0.75\, Q_o}{(<r^2>Z)} \quad \ldots\ldots\ldots\ldots\ldots\ldots(11)$$

For determining the minor (a) and the major (b) ellipsoid pivots, the following equations can be used [11].

$$a = \left[\frac{<r^2>}{3}(5 - \frac{2\delta}{0.3})\right]^{1/2} \quad \ldots\ldots\ldots\ldots\ldots(12)$$

$$b = [5 < r^2 > -2a^2]^{1/2} \quad \ldots\ldots\ldots\ldots(13)$$

While there are two methods can be used to calculate the difference between major (b) and minor (a) of ellipsoid axises [11] :

$$\Delta R_1 = \delta\, R_o \quad \ldots\ldots\ldots\ldots.(14)$$

$$\Delta R_2 = b - a \quad \ldots\ldots\ldots\ldots\ldots(15)$$

Result and Discussion:

It was focused in this study on the deformation parameters calculation for even-even ($_{58}$Ce and $_{60}$Nd) nuclei , which have neutrons magic number (N=82------- 94) for $_{58}$Ce and (N=80-------94) for $_{60}$Nd .

Two different methods for the deformation parameters calculation were applied in this study. The first method was quadrupole deformation parameter (β_2) that depended on the reduced transmission possibilities *B(E2)*↑ while the second method was the quadrupole deformation parameters (δ) that depended on the quadrupole moment Q_o.

In order to be able to compare those methods ,(β_2) was plotted as deformation parameters and (δ) as a function of neutron numbers which are demonstrated in fig.(2) and fig.(3) for ($_{58}$Ce and $_{60}$Nd) nuclei respectively. These figures showed a decreasing in the deformation parameters when they are closer to the neutrons magic number, it can also said that nuclei with neutron number (N) far from a magic number can generally be deformed. This means that nuclei with magic numbers of neutrons have a "closed shell" that fosters a spherical shape , it can also shown clearly through these mentioned figures that the values of (β_2) are larger than (δ) those for all our results, this means, the deformation that comes from transition probability $B(E2)\uparrow$ is larger than the one which comes from (Q_o). The reason for the difference between these values (β_2 and δ) is based on presumption that the values (δ) have no such nucleus surface vibrations, but values (β_2) have that , in other side the values (β_2) are affected not only by static nucleus deformation which depends on its shape, but also by the dynamic nucleus deformation that came from its surface vibration .

From tables (1) and (2) the results show obviously the difference between theoretical values for $B(E2)\uparrow$ and β_2[6] and the results from the (present work) of $B(E2)\uparrow$and β_2, this difference is caused by using "Global Best Fit" equation for calculating $B(E2)\uparrow$ which is used to calculate the deformation parameter (β_2) ,while the reference data corresponds to adopted values of $B(E2)\uparrow$.

The relationship between two deformation parameters (β_2 ,δ) was studied and led to (δ=0.9276 β_2) which showed very small difference between these two parameters. It can be noticed clearly from figures that there is an rapprochement between (β_2) and (δ) values for each isotopes of ($_{58}$Ce and $_{60}$Nd) nuclei.

In this study ,the calculation for root mean square radii $<r^2>^{1/2}$ and quadrupole moment (Q_o) was checked by making a comparison with theoretical values of ($<r^2>^{1/2}$) and (Q_o) for ($_{58}$Ce and $_{60}$Nd) nuclei ,tables (3) and (4) show this comparison respectively. As a result from this comparison, it was clearly noticed a good agreement for values for ($_{58}$Ce and $_{60}$Nd)nuclei and another good agreement for the measured and reference values of $<r^2>^{1/2}$.

Tables (5) and (6) show the results of minor (a) and major (b) ellipsoid axises for ($_{58}$Ce and $_{60}$Nd) nuclide respectively, from the results of (a & b) we calculate the difference between them (ΔR) which help as to determine the shape of nucleus (prolate, oblate or spherical). When the values of $\Delta R_2 \geq 0.85$ its shape will be prolate, $0.75 \leq \Delta R_2 < 0.85$ the nuclei will have oblate shape, while for values of $\Delta R_2 < 0.75$ the nuclei will be spherical shape.

Table (1): Mass number A, neutron number N, transition gamma energy E_{γ_0} , reduced transition probabilities *B(E2)* ↑ e^2b^2 and deformation parameters β_2 for $_{58}$Ce isotopes.

A	N	E_{γ_0} (KeV) [12]	Theoretical values [14]		Present work		
			B(E2)↑ e^2b^2 Adopted values	β_2	*B(E2)*↑ e^2b^2	R_o^2 (b)	β_2
140	82	1596.227	0.298	0.1015	0.2032	0.3883	0.0839
142	84	641.286	0.480	0.1277	0.5011	0.3919	0.1304
144	86	397.441	0.83	0.166	0.801	0.3956	0.1634
146	88	258.46	1.14	0.193	1.2205	0.3993	0.1998
148	90	158.468	1.96	0.251	1.9726	0.4029	0.2518
150	92	97.1	3.3	0.32	3.1907	0.4065	0.3173
152	94	81.7	---------	-----	3.7588	0.4101	0.3414

Table (2): Mass number A, neutron number N, transition gamma energy E_{γ_0} , reduce transition probabilities *B(E2)* ↑e^2b^2 and deformation parameter β_2 for $_{60}$Nd isotopes.

A	N	E_{γ_0} (KeV) [12]	Theoretical values [14]		Present work		
			B(E2)↑ e^2b^2 Adopted values	β_2	*B(E2)*↑ e^2b^2	R_o^2 (b)	β_2
140	80	773.73	-------	------	0.4487	0.3883	0.1204
142	82	1575.83	0.265	0.0917	0.2182	0.3919	0.0832
144	84	696.513	0.491	0.1237	0.4891	0.3956	0.1234
146	86	453.77	0.760	0.1524	0.7439	0.3993	0.1508
148	88	301.702	1.35	0.2013	1.1088	0.4029	0.1825
150	90	130.21	2.76	0.2853	2.5463	0.4065	0.274
152	92	72.51	4.2	0.349	4.5323	0.4101	0.3624
154	94	70.8	-------	-------	4.6015	0.4137	0.362

Table (3): Mass number A, neutron number N, transition gamma energy E_{γ_0} , root mean square radii $<r^2>^{1/2}$, mean square radii $<r^2>$, quadrupole moment Q_o and deformation parameters δ for $_{58}$Ce isotopes .

A	N	E_{γ_0} (KeV) [12]	Theoretical values		Present work			
			$<r^2>^{1/2}$ *fm* [13]	Q_o(b) [14]	$<r^2>$ fm^2	$<r^2>^{1/2}$ *fm*	Q_o (b)	δ
140	82	1596.227	4.8771	1.731	24.4599	4.9457	1.4299	0.0756
142	84	641.286	4.9063	2.197	24.6923	4.9691	2.244	0.1175
144	86	397.441	4.9303	2.88	24.9236	4.9924	2.8377	0.1472
146	88	258.46	4.9590	3.38	25.1539	5.0154	3.5028	0.1801
148	90	158.468	4.9893	4.43	25.3831	5.0382	4.4532	0.2269
150	92	97.1	------	5.7	25.6112	5.0608	5.6636	0.286
152	94	81.7	------	------	25.8384	5.0831	6.1471	0.3076

Table (4): Mass number A , neutron number N, transition gamma Energy E_{γ_0}, root mean square radii $<r^2>^{1/2}$, mean square radii $<r^2>$, quadrupole moment Q_o and deformation parameters δ for $_{60}$Nd isotopes.

A	N	E_{γ_0} (KeV) [12]	Theoretical values		Present work			
			$<r^2>^{1/2}$ *fm* [13]	Q_o(b) [14]	$<r^2>$ fm^2	$<r^2>^{1/2}$ *fm*	Q_o (b)	δ
140	80	773.73	4.9101	-----	24.4599	4.9457	2.128	0.1085
142	82	1575.83	4.9123	1.632	24.6923	4.9691	1.4812	0.0750
144	84	696.513	4.9421	2.222	24.9236	4.9924	2.2175	0.1112
146	86	453.77	4.9696	2.764	25.1539	5.0154	2.7348	0.1359
148	88	301.702	4.9999	3.68	25.3831	5.0382	3.3387	0.1644
150	90	130.21	5.0400	5.267	25.6112	5.0608	5.0594	0.2469
152	92	72.51	-----	6.49	25.8381	5.0831	6.7501	0.3266
154	94	70.8	-----	-----	26.0645	5.1053	6.8014	0.3262

Table (5): Mass number A, neutrons number N, small and large ellipsoid axises (a, b) and the difference between them ΔR by two methods, for $_{58}$Ce isotopes.

A	**N**	**c**			
		a (*fm*)	**B(*fm*)**	**Δ R_1(*fm*)**	**Δ R_2 *(fm)***
140	82	2.7226	3.147	0.4198	0.4245
142	84	2.6427	3.2981	0.6561	0.6554
144	86	2.5860	3.4040	0.8257	0.8181
146	88	2.5203	3.5175	1.0146	0.9972
148	90	2.4201	3.6711	1.284	1.251
150	92	2.2845	3.8557	1.6257	1.5712
152	94	2.2354	3.9271	1.7567	1.6917

Table (6): Mass number A, neutrons number N, small and large ellipsoid axises (a, b) and the difference between them ΔR by two methods, for $_{60}$Nd isotopes.

A	N	c			
		a (*fm*)	b (*fm*)	Δ R_1 (*fm*)	Δ R_2(*fm*)
140	80	2.6552	3.2601	0.6030	0.6050
142	82	2.7302	3.1524	0.4186	0.4223
144	84	2.6621	3.2846	0.6237	0.6225
146	86	2.6162	3.3746	0.7657	0.7585
148	88	2.5605	3.4754	0.9306	0.9149
150	90	2.3786	3.7401	1.4039	1.3616
152	92	2.1871	3.9811	1.8647	1.7941
154	94	2.1928	3.9887	1.8707	1.7960

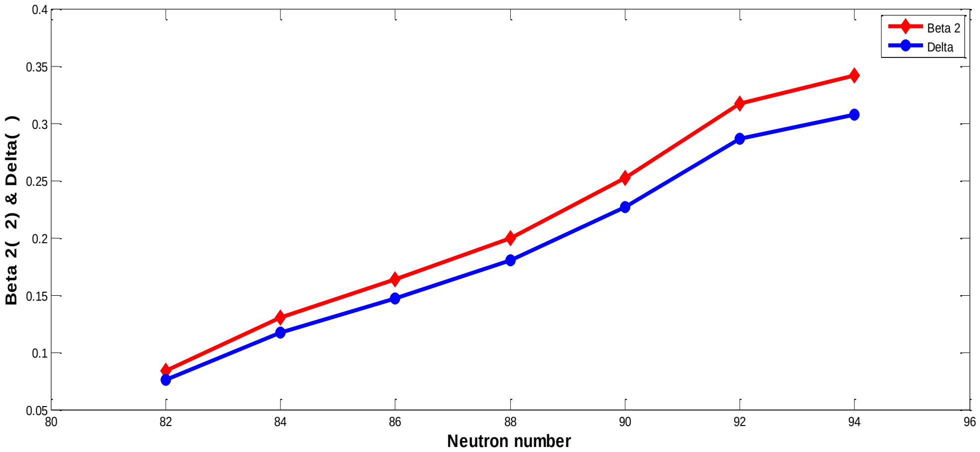

Fig.(2): comparison between β_2 and δ values as function of neutrons number for $_{58}$Ce isotopes .

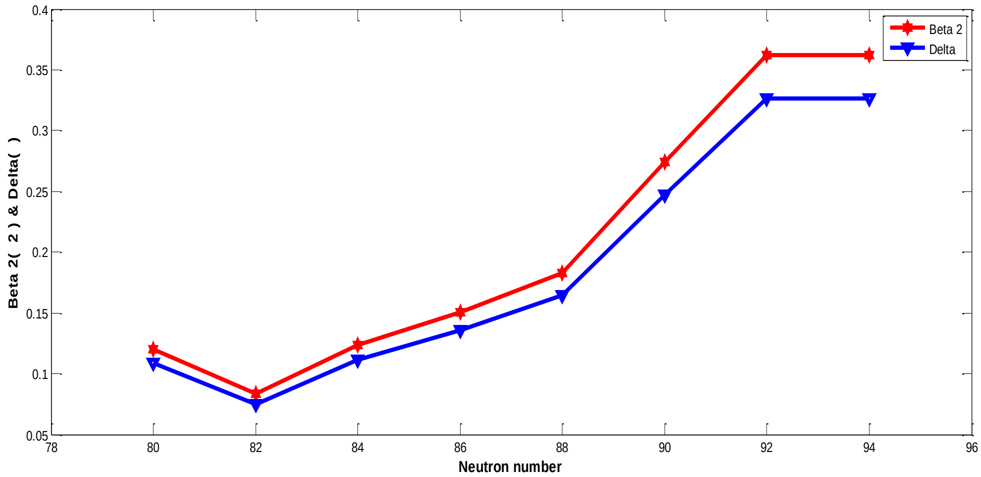

Fig.(3): comparison between β_2 and δ values as function of neutrons number for $_{60}$Nd isotopes .

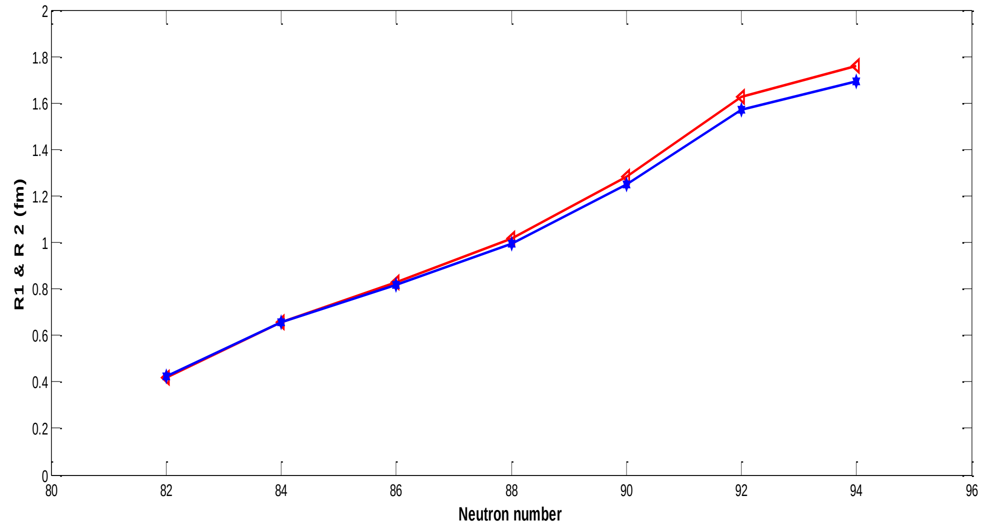

Fig. (4): The difference between small and large ellipsoid axises

(a, b) R1 and R2 by two methods, for $_{58}$Ce isotopes.

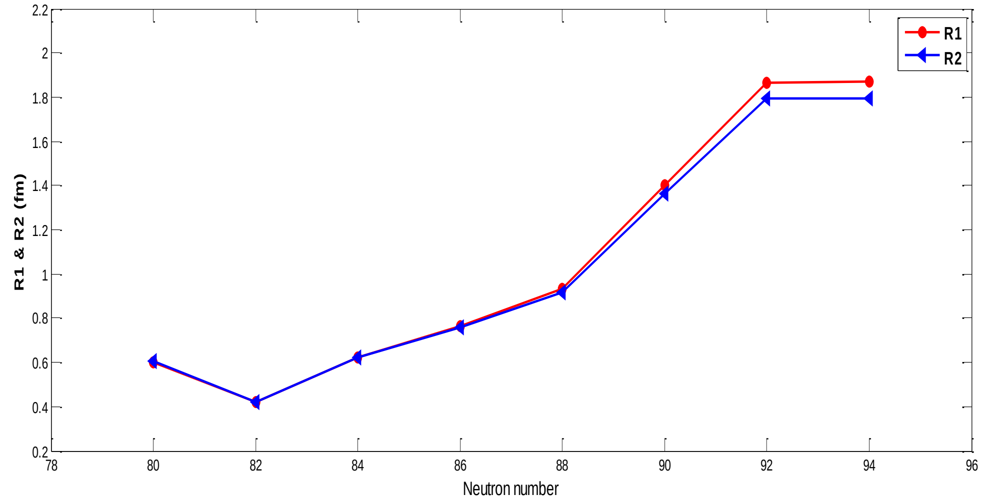

Fig. (5): The difference between small and large ellipsoid axises

(a, b) R1 and R2 by two methods, for $_{60}$Nd isotopes.

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