A Note on Multipliers in GK Algebra

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***Abstract*: In this paper, we introduce the concept of multipliers in GK algebra and also we discuss about the properties of the regular multiplier of GK algebra. We also introduce the kernel of multipliers of GK algebra.**

I.INTRODUCTION

In 1971, R.Larsen [3] introduced the theory of multipliers. In continuation of this, in 1980 W.H.Cornish[5] introduced the concept of multipliers in implicative BCK algebras. After that many researchers have applied this concept in their algebraic structure and brought some interesting properties of multipliers. Motivated by their work ,in this paper we discuss about multiplier in GK algebra[2] and Kernel of multiplier in GK algebra and also discuss some properties of regular multiplier of GK algebra.

II.MULTIPLIERS IN GK ALGEBRA

2.1 Definition

Let $(\mathcal{X},*,1)$ *be an GK algebra. A self map* Δ *is called a right multipliers of* $\mathcal{X}$ *if* $\Delta(m*n) = m*\Delta(n)$ *for all* $m,n \in \mathcal{X}$.

2.2 Example

Consider $\mathcal{X} = \{1,2,3\}$ in which '$*$' is defined by

$*$	1	2	3
1	1	3	2
2	2	1	3
3	3	2	1

Then $\mathcal{X}$ is an GK algebra.

Define a mapping $\Delta: \mathcal{X} \to \mathcal{X}$ by

$$\Delta(\mathrm{m}) = \begin{cases} 1 & if \;\; x = 1 \\ 2 & if \; x = 2 \\ 3 & if \; x = 3 \end{cases}$$

It is clearly known that Δ is a right multiplier of GK algebra.

2.3 Definition

Let $(\mathcal{X}, *, 1)$ be an GK algebra. A self map Δ is called a left multipliers of $\mathcal{X}$ if $\Delta(m * n) = \Delta(m) * n$ for all $m, n \in \mathcal{X}$.

Note: The above said example is also an example of the left multiplier of GK algebra.

2.4 Definition:

A map Δ of an GK algebra $\mathcal{X}$ is said to be regular if $\Delta(1) = 1$.

2.5 Proposition

Let Δ be a left multiplier of $\mathcal{X}$, then

(i) For every m in $\mathcal{X}$, $\Delta(1) = \Delta(m) * m$.
(ii) Δ is 1-1.

Proof:

(i) Let $\mathrm{m} \in \mathcal{X}$. Then $m * m = 1$.
We have $\Delta(1) = \Delta(\mathrm{m} * \mathrm{m}) = \Delta(m) * m$ for all $m \in \mathcal{X}$.
(ii) Let $m, n \in \mathcal{X}$ such that $\Delta(m) = \Delta(n)$.
Then by (i), we have $\Delta(1) = \Delta(\mathrm{m} * \mathrm{m}) = \Delta(m) * m$ and $\Delta(1) = \Delta(\mathrm{n} * \mathrm{n}) = \Delta(n) * n$.
Then $\Delta(m) * m = \Delta(n) * n$.
By cancellation law, $m = n$.
$\therefore \; \Delta$ $is\ 1 - 1$

2.6 Proposition

Let Δ be a right multiplier of $\mathcal{X}$, then

(i) For every m in $\mathcal{X}$, $\Delta(1) = m * \Delta(m)$.
(ii) Δ is 1-1.

Proof:

(i) Let $\mathrm{m} \in \mathcal{X}$. Then $m * m = 1$.

We have $\Delta(1) = \Delta(m * m) = m * \Delta(m)$ *for all* $m \in \mathcal{X}$.

(ii) Let $m, n \in \mathcal{X}$ such that $\Delta(m) = \Delta(n)$.

Then by (i), we have $\Delta(1) = \Delta(m * m) = m * \Delta(m)$ and

$\Delta(1) = \Delta(n * n) = n * \Delta(n)$. Then $m * \Delta(m) = n * \Delta(n)$.

By cancellation law, $m = n$.

$\therefore \Delta$ *is* $1 - 1$.

2.7 Theorem:

Let Δ be a left multiplier of $\mathcal{X}$. Then $\Delta(m) = m$ iff Δ is regular.

Proof:

Let Δ is regular. Since $\Delta(1) = 1$.

Then we have $\Delta(1) = \Delta(m * m) = \Delta(m) * m = 1$.

By definition of GK algebra, $\Delta(m) = m$.

Conversely, let $\Delta(m) = m$ *for* m *in* $\mathcal{X}$.

It is clear that $\Delta(1) = 1$.

Hence Δ is regular.

2.8 Proposition

Let $\mathcal{X}$ be GK algebra and let Δ be a left multiplier of $\mathcal{X}$.

If $\Delta(m) * m = 1$ *for every* $\mathcal{X}$, then Δ is regular.

Proof:

Let $\Delta(m) * m = 1$ and let Δ be a left multiplier of $\mathcal{X}$.

By definition of GK algebra,

We have $\Delta(1) = \Delta(m * m) = \Delta(m) * m = 1$.

Hence Δ is regular.

2.9 Proposition

Let Δ be a left multiplier of $\mathcal{X}$. Then the following holds

(i) If $\exists$ an element $m \in \mathcal{X} \ni$: $\Delta(m) = m$, Δ is the identity.

(ii) If $\exists$ an element $m \in \mathcal{X} \ni$: $\Delta(n) * m = 1$ *for every* $n \in \mathcal{X}$ then $\Delta(n) = m$.

Proof:

(i) Let $\Delta(m) = m$ for some $m \in \mathcal{X}$.
Then $\Delta(m) * m = m * m$
$\Rightarrow \Delta(m) * m = 1$.
Hence $\Delta(1) = 1$ by the proposition(2.7)
Which implies that Δ is regular.

(ii) By the definition of GK algebra,
$\Delta(\mathrm{m} * \mathrm{n}) = \Delta(\mathrm{n} * \mathrm{m}) = \Delta(1)$
$\Rightarrow \Delta(m) * n = \Delta(n) * m = \Delta(1)$
$\Rightarrow \Delta(m) * n = 1$
$\Rightarrow \Delta(m) = n$.

2.10 Proposition

Let $\mathcal{X}$ be a GK algebra and Δ be a left multiplier of $\mathcal{X}$. Then

$\Delta(\Delta(m) * m) = 1 \; \forall \, m \in \mathcal{X}$

Proof:

Let $m \in \mathcal{X}$. Then we have $\Delta(\Delta(m) * m) = \Delta(m) * \Delta(m) = 1$.

2.11 Proposition

Let $\mathcal{X}$ be a GK algebra and let Δ be a regular multiplier. Then the self map Δ is an identity map if it satisfies left multiplier is equal to right multiplier that is $\Delta(m) * n = m * \Delta(n) \; \forall \, m, n \in \mathcal{X}$.

Proof:

Since Δ is regular, we have $\Delta(1) = 1$.

Let $\Delta(m) * n = m * \Delta(n) \; \forall \, m, n \in \mathcal{X}$

Then $\Delta(m) = \Delta(m * 1) = \Delta(m) * 1 = m * \Delta(1) = m * 1 = m$.

Hence Δ is an identity map.

2.12 Definition

Let Δ be a multiplier of GK algebra. A set defined by $\mathcal{H}_\Delta(\mathcal{X})$ by

$\mathcal{H}_\Delta(\mathcal{X}) = \{m \in \mathcal{X} \,/\, \Delta(m) = m\} \; \forall \, m \in \mathcal{X}$.

2.13 Proposition

Let $\mathcal{X}$ be a GK algebra and let Δ be a left multiplier on $\mathcal{X}$. If $n \in \mathcal{H}_\Delta(\mathcal{X})$, we have $m \wedge n \in \mathcal{H}_\Delta(\mathcal{X}) \; \forall \, m, n \in \mathcal{X}$.

Proof:

Let Δ be a left multiplier on $\mathcal{X}$ and let $n \in \mathcal{H}_\Delta(\mathcal{X})$.

Now $\Delta(m \wedge n) = \Delta(n * (n * m))$

$= \Delta(n) * (n * m)$

$= n * (n * m)$

$= m \wedge n.$

Hence $m \wedge n \in \mathcal{H}_\Delta(\mathcal{X})$.

2.14 Proposition

Let $\mathcal{X}$ be a GK algebra and let Δ be a right multiplier on $\mathcal{X}$. If $n \in \mathcal{H}_\Delta(\mathcal{X})$, we have $m \wedge n \in \mathcal{H}_\Delta(\mathcal{X})\ \forall\, m, n \in \mathcal{X}$.

Proof:

Let Δ be a right multiplier on $\mathcal{X}$ and let $n \in \mathcal{H}_\Delta(\mathcal{X})$.

Now $\Delta(m \wedge n) = \Delta(n * (n * m))$

$= n * \Delta(n * m)$

$= n * (n * \Delta(m))$

$= n * (n * m)$

$= m \wedge n.$

Hence $m \wedge n \in \mathcal{H}_\Delta(\mathcal{X})$.

2.15 Definition

Let $\mathcal{X}$ be an GK algebra and Δ_1, Δ_2 two self maps. We define a mapping

$\Delta_1 \circ \Delta_2 : \mathcal{X} \to \mathcal{X}$ *by* $(\Delta_1 \circ \Delta_2)(m) = \Delta_1(\Delta_2(m))\ \forall\, m \in \mathcal{X}$.

2.16 Proposition

Let $\mathcal{X}$ be an GK algebra and Δ_1, Δ_2 two right (left) multipliers of $\mathcal{X}$. The $\Delta_1 \circ \Delta_2$ is also right (left) multiplier of $\mathcal{X}$.

Proof:

Let $\mathcal{X}$ be an GK algebra and Δ_1, Δ_2 two right multipliers of $\mathcal{X}$. Then we have

$(\Delta_1 \circ \Delta_2)(m * n) = \Delta_1(\Delta_2(m * n))$

$= \Delta_1(m * \Delta_2(n))$

$= m * \Delta_1(\Delta_2(n))$

$= m * (\Delta_1 \circ \Delta_2)(n)$

Let $\mathcal{X}$ be an GK algebra and Δ_1, Δ_2 two left multipliers of $\mathcal{X}$. Then we have

$(\Delta_1 \circ \Delta_2)(m * n) = \Delta_1(\Delta_2(m * n))$

$= \Delta_1(\Delta_2(m)) * n$

$= (\Delta_1 \circ \Delta_2)(m) * n.$

2.17 Definition

Let $\mathcal{X}$ be an GK algebra and Δ_1, Δ_2 two self maps. We define $(\Delta_1 \wedge \Delta_2): \mathcal{X} \to \mathcal{X}$ by $(\Delta_1 \wedge \Delta_2)(m) = \Delta_1(m) \wedge \Delta_2(m)$.

2.18 Proposition

Let $\mathcal{X}$ be an GK algebra and Δ_1, Δ_2 two left multiplier of $\mathcal{X}$. Then $\Delta_1 \wedge \Delta_2$ is also left multiplier of $\mathcal{X}$.

Proof:

Let $\mathcal{X}$ be an GK algebra and Δ_1, Δ_2 two multiplier of $\mathcal{X}$.

$(\Delta_1 \wedge \Delta_2)(m * n) = \Delta_1(m * n) \wedge \Delta_2(m * n)$

$= (\Delta_1(m) * n) \wedge (\Delta_2(m) * n)$

$= (\Delta_2(m) * n) * ((\Delta_2(m) * n) * (\Delta_1(m) * n))$

$= \Delta_1(m) * n$ ……………….(1)

$(\Delta_1 \wedge \Delta_2)(m) * n = (\Delta_1(m) \wedge \Delta_2(m)) * n$

$= \Delta_1(m) * n$ ………………..(2)

From (1)and (2)

$(\Delta_1 \wedge \Delta_2)(m * n) = (\Delta_1 \wedge \Delta_2)(m) * n.$

Hence $\Delta_1 \wedge \Delta_2$ is left multiplier.

2.19 Definition

For any $\omega \in \mathcal{Q}(\mathcal{X})$, the set of all multipliers ,we define the Kernel of ω as follows

$\mathcal{K}_\omega = \{m \in \mathcal{X} / \omega(m) = 1\}.$

2.20 Proposition

Let ω be a multiplier of $\mathcal{X}$ *and* $1 - 1$.Then $\mathcal{K}_\omega$ is {1}

Proof:

Let ω be one-to-one.

Let $m \in \mathcal{K}_{\omega}$. So $\omega(m) = 1 = \omega(1)$. Thus $m = 1$.

So $\mathrm{Ker}(\omega) = \{1\}$.

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