Fuzzy ℓ-Filters Via Fuzzy ℓ- Partial Ordering

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Abstract

In this paper is based on the presentation of fuzzy ℓ- filters via fuzzy partial ordering and investigated some theorems on fuzzy ℓ- filters.

Keyword: Fuzzy ℓ-filters, fuzzy ℓ-partial ordering.

1 Introduction

The relation between logics of algebra and modern algebra was worked by many mathematicians from Boolean. The idea of a fuzzy set from the crisp set was introduced by L.A. Zadeh[6] and the study of fuzzy algebraic structures was initiated by Rosenfeld, since then various algebraic structures were converted to fuzzy algebra. The application of group theory is important in the design of fast adders and error- correcting codes, it can be used in number of applications dealing with topics such as compilation of expression in polish notation, language and grammars and for the theory of fast adders and error - detecting and correcting codes .

Lattice structure has been found to be extremely important in the areas of communication systems and information analysis. Some system models often include exessive complexity of the situation which in turn may lead to consequence where it is difficult to formulate the model or the model is too complicated to be used in practice.

Nanda.S[3] defined the concept of fuzzy lattice latter Kanakana chakraborty [2] has modified the definition for fuzzy lattice.N.K. Saha[4] has defined the concept of $\Gamma-$ semigroups and established a relation between regular Γ- semigroup and $\Gamma-$ group.K.L.N. Samy[5] has investigated dually residuated lattice ordered semigroups.

In this paper we give the new idea of fuzzy ℓ- filters via fuzzy partial ordering and develope some theorems to fuzzy ℓ- filters.

2 Preliminaries

Let χ be any set and let γ be a fuzzy relation defined over χ. Then γ is said to be Max-min transitive if $\gamma.\gamma \subseteq \gamma$ or more explicitly if $\forall(\vartheta_1, \vartheta_2, \vartheta_3) \in \chi^3$ $\mu_{\gamma(\vartheta_1, x\vartheta_3)} \geq min\{\mu_{\gamma(\vartheta_1,\vartheta_2)}, \mu_{\gamma(\vartheta_2,\vartheta_3)}\}$ Reflexive if $\forall\vartheta_1 \in \chi, \mu_{\eta(\vartheta_1,\vartheta_2)} = 1$ Perfect antisymmetric if $\forall(\vartheta_1, \vartheta_2) \in \chi^2, \vartheta_1 \neq \vartheta_2, \mu_{\gamma(\vartheta_1,\vartheta_2)} > 0 \Rightarrow \mu_{\gamma(\vartheta_2,\vartheta_1)} = 0$, where $\mu_{\gamma(\vartheta_1,\vartheta_2)}$ represent the membership value of the pair $(\vartheta_1, \vartheta_2) \in \gamma$.
The fuzzy relation $\bar{S}$ defined over a set γ is said to be fuzzy partial ordering if and only if it is reflexive, max- min transitive and perfectly antisymmetric. A set χ along with a fuzzy partial ordering $\bar{S}$ defined on it is called a fuzzy partially ordered set.
Let χ be a fuzzy partially ordered set with a fuzzy partial order $\bar{S}$ defined over it with each $\vartheta_1 \in \gamma$ we associate two fuzzy sets
The dominating class $\bar{S} \geq (\vartheta_1)(v_2) = \bar{S}(\vartheta_2, \vartheta_1)$
The dominated class $\bar{S} \leq (\vartheta_1)(\vartheta_2) = \bar{S}(\vartheta_1, \vartheta_2)$
Let ω be a non fuzzy subset of γ.
Then the fuzzy maximum of ω denoted by $U_{\phi(\omega)} = \bigcap_{\vartheta_1 \in \omega} \bar{P} \geq (\vartheta_1)$.
Then the fuzzy minimum of ω denoted by $L_{\phi(\omega)} = \bigcup_{\vartheta_1 \in \omega} \bar{P} \leq (\vartheta_1)$.

Definition 2.1 *Let $\bar{\tau}$ be a fuzzy partially ordered set and let $\bar{\sigma}$ be a fuzzy subset of $\bar{\tau}$. Then $\bar{\sigma}$ is said to be a fuzzy lattice in $\bar{\tau}$ if every pair of elements in $\bar{\tau}$ has a fuzzy minimum L_ϕ and fuzzy maximum U_ϕ, where both L_ϕ and U_ϕ are fuzzy subsets of $\bar{\tau}$ satisfying the following two conditions:*

$$\begin{aligned} \mu_{max\{U_\phi\}(\vartheta_1)} &\geq \mu_{\bar{\sigma}(\vartheta_1)}, \forall\vartheta_1 \in \bar{\tau} \\ \mu_{min\{L_\phi\}(\vartheta_1)} &\geq \mu_{\bar{\sigma}(\vartheta_1)}, \forall\vartheta_1 \in \bar{\tau} \end{aligned}$$

Definition 2.2 *A non-empty subset $\boldsymbol{F}$ of a lattice $\boldsymbol{L}$ is called a filter if*
(i) $\vartheta \in \boldsymbol{F}, \varsigma \in \boldsymbol{F} \Longrightarrow \vartheta \cap \varsigma \in \boldsymbol{F}$
(ii) $\vartheta \in \boldsymbol{F}, \varsigma \in \boldsymbol{L}$ and $\varsigma \geq \vartheta \Longrightarrow \varsigma$ in $\boldsymbol{F}$.

3 Fuzzy ℓ-Filters Via Fuzzy ℓ- Partial Ordering

Definition 3.1 *Let $\boldsymbol{F}$ be a fuzzy lattice,a fuzzy subset μ of $\boldsymbol{F}$ is said to be a fuzzy ℓ-Filters on $\boldsymbol{F}$, if it satisfies the following axioms*
(i) $\mu_{\bar{\omega}(0)} \leq \mu_{\bar{\omega}(\vartheta)}$
(ii) $\mu_{\bar{\omega}(\vartheta)} \leq \mu_{minL_\phi(\vartheta)}$.

Example 3.1 *Let us consider $(Z_4, *)$ and let $\mu_{\bar{\omega}(\vartheta)} = \{0, .8, 0, .5\}$*

*	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

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We have lower bounds for $\{0, 1, 2, 3\}$

$$\begin{aligned} L_{\phi(0,1)} &= \{0, 1, 2, 3\} \\ L_{\phi(0,2)} &= \{0, 2, 0, 2\} \\ L_{\phi(0,3)} &= \{0, 3, 2, 1\} \\ L_{\phi(1,2)} &= \{0, 2, 2, 3\} \\ L_{\phi(1,3)} &= \{0, 3, 2, 3\} \\ L_{\phi(2,3)} &= \{0, 3, 2, 2\} \end{aligned}$$

Therefore $minL_\phi(x) = \{0, 1, 0, 1\}$
$\mu_{\bar{\omega}(\vartheta)} \leq \mu_{minL_\phi(\vartheta)}$.
Hence $(Z_4, *)$ *is a fuzzy* ℓ*-filter.*

Definition 3.2 *Let* α *and* β *be two fuzzy* ℓ*- filters in* $\boldsymbol{F}$*. Then*
(i) $\mu_{\bar{\alpha}}(\vartheta) = \mu_{\bar{\beta}}(\vartheta)$ *iff* $\bar{\alpha} = \bar{\beta}$*, for all* $\vartheta \in \boldsymbol{F}$*.*
(ii) $\mu_{\bar{\alpha}}(\vartheta) \subseteq \mu_{\bar{\beta}}(\vartheta)$ *iff* $\bar{\alpha} \subseteq \bar{\beta}$*, for all* $\vartheta \in \boldsymbol{F}$*.*
(iii) $\mu_{\bar{\gamma}} = min\{\bar{\alpha}(\vartheta), \bar{\beta}(\vartheta)\}$ *iff* $\bar{\gamma} = \bar{\alpha} \cap \bar{\beta}$*.*

Definition 3.3 *A fuzzy set* μ *in* $\boldsymbol{F}$ *is called fuzzy* ℓ *-filter* $\boldsymbol{F}$ *if it satifies the following inequalities*
(i) $\mu_{\bar{\alpha}(0)} \leq \mu_{\bar{\alpha}(\vartheta)}$
(ii) $\mu_{\bar{\alpha}(\vartheta)} \leq \mu_{minL_\phi(\vartheta)}$
(iii) $\{\mu_{\bar{\alpha}}(\vartheta), \mu_{\bar{\beta}}(x)\} \leq min\{L_\phi(\vartheta)\}$

Definition 3.4 *If* α *and* β *be two fuzzy sets* $\boldsymbol{F}$*. The cartesian product* $\alpha \times \beta : \Sigma \times \Sigma \longrightarrow [0, 1]$ *is defined by* $(\alpha \times \beta)(\vartheta, \varsigma) = min\{L_\alpha(\vartheta), L_\beta(\varsigma)\}$*, where* $\{L_\alpha(\vartheta), L_\beta(\varsigma)\}$ *are greatest least element of* $\boldsymbol{F}$*.*

Theorem 3.1 *If* α *and* β *be two fuzzy* ℓ *- filters of* $\boldsymbol{F}$*. Then* $\alpha \times \beta$ *is a fuzzy* ℓ*-filter of* $\vartheta \times \vartheta$*.*

Definition 3.5 *Let* $\xi : \eta \longrightarrow \gamma$ *be a mapping of fuzzy* $\ell-$ *filter and* η *be a fuzzy set of* γ*. Then the mapping* γ_ξ *is the preimage of* γ *if* $\gamma_{\xi(\vartheta)} = \gamma\xi(\vartheta)$*, for all* $\vartheta \in \eta$*.*

Theorem 3.2 *Let* $\xi : \eta \longrightarrow \gamma$ *be a homomorphism. If* γ *is a fuzzy* ℓ *filter of* ξ *, then* γ_ξ *is a fuzzy* ℓ*- filter of* η*.*

Theorem 3.3 *Let* $\xi : \eta \longrightarrow \gamma$ *be an epimorphism of fuzzy* $\ell-$ *filter. If* γ_η *is a fuzzy* ℓ*- filter of* η*, then* ξ *is a fuzzy* ℓ*- filter in* γ*.*

Conclusion So far in research algebraic structures were converted to lattice algebraic structure, in this paper we convert fuzzy ℓ- filter via fuzzy partial ordering. In the same way it will be interesting to convert other fuzzy lattice structure via fuzzy partial ordering.

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