On the Conformal curvature tensor of Sasakian manifolds admitting Zamkovoy connection

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Abstract

The purpose of this paper is to study some properties of Sasakian Manifold admitting Zamkovoy connection D^a . Moreover we study Conformal curvature tensor with respect to D^a and showed that a conformally flat, Sasakian manifold is an 4–Einstein manifold and a Sasakian manifold is locally conformally $-symmetric with respect to Zamkovoy connection D^a if and only if it is so with respect to the Levi-Civita connection .Also a sasakian manifold satisfying C^a (Ⓐ,) R^a = O.is an 4 Einstein manifold ,where C^a and R^a are conformal Curvature tensor and Riemannian Curvature tensor with respect to D^a respectively.

1 Imtroductiom

The Weyl conformal curvature tensor C of rank 4 of an n-dimensional Riemannian Manifold M,(n > 3) is defined by

$$\begin{aligned} & C(E,Y,Z,W) \\ = \ & R(E,Y,Z,W) - \frac{fi}{n-2}[S(Y,Z)g(E,W) - S(E,Z)g(Y,W) \\ & ‡g(Y,Z)g(ⓟE,W) - g(E,Z)g(ⓟY,W)] \\ & ‡\frac{v}{(n-fi)(n-2)}[g(Y,Z)g(E,W) - g(E,Z)g(Y,W)] \qquad (fi) \end{aligned}$$

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for all E, Y, Z & W z(M), set of all vector fields of the manifold M, where C (E, Y, Z, W) denotes the conformal curvature tensor of type (0, 4) and R (E, Y, Z, W) denotes the Riemannian curvature tensor of type (0, 4) defined by

$$C(E, Y, Z, W) = g(C(E, Y)Z, W) \quad (2)$$

$$R(E, Y, Z, W) = g(R(E, Y)Z, W), \quad (3)$$

where R (E, Y) Z is the Riemannian tensor of type (0, 3), C (E, Y) Z is the conformal curvature tensor of type (0, 3) and S denotes the Ricci tensor of type (0, 2) and v is the scalar curvature. The curvature tensor defined in equation (fi) is known as conformal curvature tensor.

In fi924, the idea of semi-symmetric linear connection on a differentiable manifold was introduced by A. Friedmann and J. A. Schouten [fi] . In fi9F0, K. Yano [3] started a systematic study of the semi-symmetric metric connection in a Riemannian manifold and this was further studied by various authors.In CR Geometry N.Tanaka [F] and S.M Webster[6] introduced independently a new connection.As a generalization of the above two connection S.Tanno introduced a new connection called Tanaka-Webster connection In a paracontact metric manifold S.Zamkovoy[†] defined a canonical connection , known as Zamkovoy connection,which plays the same role of the (generalized) Tanaka-Webster connection[8] in paracontact geometry.

For an n-dimensional almost contact metric manifold M equipped with an almost contact metric structure ($, Ⓐ, 4, g) consisting of a (fi, fi) tensor field $, a vector field Ⓐ, a fi-form 4 and a Riemannian metric g, S. Zamkovoy has introduced a new connection known as Zamkovoy connection ð^a , which is related with Levi-civita connection ð as

$$ð^a_X Y = ð_X Y ‡ (ð_X 4)(Y) Ⓐ — 4(Y) ð_X Ⓐ ‡ 4(E) \$Y, \quad (4)$$

for all E, Y z(M) .

In a Sasakian manifold M of dimension (n > 2), the conformal curvature tensor C^a with respect to the Zamkovoy connection ð^a is given by

$$C^a(E, Y)Z = R^a(E, Y)Z — \frac{fi}{n — 2}[S^a(Y, Z)E — S^a(E, Z)Y ‡ g(Y, Z)ⓟ^a E — g(E, Z)ⓟ^a Y] ‡ \frac{v}{(n — fi)(n — 2)}[g(Y, Z)E — g(E, Z)Y] \quad (†)$$

where R^a and S^a are Riemannian curvature tensor and Ricci tensor with respect to Zamkovoy connection ð^a respectively.

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$$S(V, W) = h_{fi}(V, W) ‡ h_2 4(V) 4(W)$$

fov a11 U, V c z(M), set of a11 seGtov fle1ds of the mansfo1d M and h_{fi} and h_2 ave sGa1avs.

This paper is organised as follows:

After introduction ,a short description of Sasakian manifold is given in section 2. In section 3 ,we have discussed Sasakian manifold admitting Zamkovoy connection ð and obtained curvature tensor R^a,Ricci tensor S^a ,Scalar curvature tensor v^a,Ricci operator Ⓟ^{a} with respect to ^{a} ð in Sasakian manifold. In sction 4 . we deals with conformally flat Sasakian manifold with respect to the connection ð^a and we find that conformally flat Sasakian manifold with respect to ð is an 4 Einstein manifold . In section † we showed Sasakian manifold M, is locally conformally $-symmetric with respect to Zamkovoy connection ð^a if and only if it is so with respect to the Levi-Civita connection ð. Finally in section 6 we discussed a Sasakian manifold satisfying C^a (Ⓐ, U) ◦ R^a = 0 .

2 Prelimimaries

Let M be an n-dimensional almost contact metric manifold equipped with an almost contact metric structure ($, Ⓐ, 4, g) consisting of a (fi, fi) tensor field $, a vector field Ⓐ, a fi-form 4 and a Riemannian metric g. Then

$$\$^2 Y = -Y ‡ 4(Y)Ⓐ, 4(Ⓐ) = fi, 4(\$E) = 0, \ \$Ⓐ = 0, \quad (6)$$

$$g(\$E, \$Y) = g(E, Y) — 4(E)4(Y), \quad (F)$$

$$g(E, \$Y) = -g(\$E, Y), 4(Y) = g(Y, Ⓐ), 6E, Y \ c \ z(M) \quad (8)$$

An almost contact metric manifold M is said to be

(i) a contact metric manifold if

$$g(E, \$Y) = d4(E, Y), 6E, Y \ c \ z(M) \ ¡ \quad (9)$$

(ii) a K-contact manifold if the vector field Ⓐ is Killing equivalently

$$ð_× Ⓐ = -\$E, \quad (fi0)$$

where ð is Riemannian connection

(iii) a Sasakian manifold if

$$(ð_× \$)Y = g(E, Y)Ⓐ - 4(Y)E, 6E, \ Y \ c \ z(M). \quad (fifi)$$

A K-contact manifold is a contact metric manifold, while the converse is true if the Lie derivative of $ in the characteristic direction Ⓐ vanishes identically. A Sasakian manifold is always a K-contact manifold. A 3-dimensional K-contact manifold is a Sasakian manifold.

fi. It is well known that a contact metric manifold is Sasakian if and only if

$$R(E, Y)Ⓐ = 4(Y)E — 4(E)Y, 6E, Y \ c \ z(M). \quad (fi2)$$

In a Sasakian manifold equipped with the structure ($, Ⓐ, 4, g), the following relations also hold ([9],[4], [fifi]):

(ðX4) Y = g (E, $Y) , (fi3)

R(Ⓐ, E)Y = g(E, Y)Ⓐ — 4(Y)E, (fi4)

S (E, Ⓐ) = (n — fi) 4 (E) , (fi†)

R (E, Ⓐ) Y = 4 (Y) E — g (E, Y) Ⓐ, (fi6)

ⓅⒶ = (n — fi) Ⓐ (fiF)

S (E, Y) = g (ⓅE, Y) (fi8)

R(E, Y)Z = g(Y, Z)E — g(E, Z)Y (fi9)

Σ S (es,$es) = 0, Σ S (es,es) = v(sca1av cuvvatuve) (20)

Σ g ($es,$es) = n — fi, Σ g (es,$es) = 0 (2fi)

Where the summation is taken over s from fi to n.

3 Some properties of Sasakiam mamifold admitting Eamkovoy commectiom

In Sasakian manifold, using (fi0) and (fi3) equation (fi) reduces to

ðaXY = ðXY ‡ g (E, $Y) Ⓐ ‡ 4 (Y) $E ‡ 4 (E) $Y. (22)

Putting Y = Ⓐ in (22)

ðaXⒶ = 0. (23)

Now by the help of (22), (fi0) and (fifi) we get the followings

ðaX4 (Y)
= ðaX g (Y, Ⓐ)
= 4 (ðXY) ‡ g (E, $Y) , (24)

ðaX ($Y)
= ðX ($Y) — g ($E, $Y) Ⓐ — 4 (E) Y ‡ 4 (E) 4 (Y) Ⓐ, (2†)

ðaXg (Y, $Z)
= g (ðXY, $Z) ‡ 4 (E) g ($Y, $Z) ‡ g (Y, ðX ($Z))
—4 (E) g (Y, Z) ‡ 4 (E) 4 (Y) 4 (Z) . (26)

Now we know that

Ra (E, Y) Z = ðaX ðaY Z — ðaY ðaXZ — ða[X,Y]Z. (2F)

By using (22), (23), (24), (2†) and (26) we obtain the followings

ðᵃXðᵃY Z
= ðXðY Z ‡ g (E, $ðY Z) Ⓐ ‡ 4 (ðY Z) $E ‡ 4 (E) $ðY Z
‡g (ðXY, $Z) Ⓐ ‡ 4 (E) g ($Y, $Z) Ⓐ ‡ g (Y, ðX ($Z)) Ⓐ
—4 (E) g (Y, Z) Ⓐ ‡ 4 (E) 4 (Y) 4 (Z) Ⓐ — g (Y, $Z) $E
‡g (Y, $Z) $E ‡ 4 (ðXZ) $Y ‡ g (E, $Z) $Y
‡g (Z, $E) $Y ‡ 4 (Z) ðX ($Y) — 4 (Z) g ($E, $Y) Ⓐ
—4 (Z) 4 (E) Y ‡ 4 (Z) 4 (E) 4 (Y) Ⓐ ‡ 4 (ðXY) $Z
‡g (E, $Y) $Z — g (Y, $E) $Z ‡ g (Y, $E) $Z
‡4 (Y) ðX ($Z) — 4 (Y) g ($E, $Z) Ⓐ — g (Z, $E) $Y
—4 (Y) 4 (E) Z ‡ 4 (Y) 4 (E) 4 (Z) Ⓐ, (28)

and

ðᵃ[X,Y]Z
= ð[X,Y]Z ‡ g (ðXY, $Z) Ⓐ — g (ðY E, $Z) Ⓐ ‡ 4 (Z) $ðXY
—4 (Z) $ðY E ‡ 4 (ðXY) $Z — 4 (ðY E) $Z. (29)

Interchanging E and Y in (28)

ðᵃY ðᵃXZ
= ðY ðXZ ‡ g (Y, $ðXZ) Ⓐ ‡ 4 (ðXZ) $Y ‡ 4 (Y) $ðXZ
‡g (ðY E, $Z) Ⓐ ‡ 4 (Y) g ($E, $Z) Ⓐ ‡ g (E, ðY ($Z)) Ⓐ
—4 (Y) g (E, Z) Ⓐ ‡ 4 (Y) 4 (E) 4 (Z) Ⓐ — g (E, $Z) $Y
‡g (E, $Z) $Y ‡ 4 (ðY Z) $E ‡ g (Y, $Z) $E
‡g (Z, $Y) $E ‡ 4 (Z) ðY ($E) — 4 (Z) g ($Y, $E) Ⓐ
—4 (Z) 4 (Y) E ‡ 4 (Z) 4 (Y) 4 (E) Ⓐ ‡ 4 (ðY E) $Z
‡g (Y, $E) $Z — g (E, $Y) $Z ‡ g (E, $Y) $Z
‡4 (E) ðY ($Z) — 4 (E) g ($Y, $Z) Ⓐ — g (Z, $Y) $E
—4 (E) 4 (Y) Z ‡ 4 (E) 4 (Y) 4 (Z) Ⓐ, (30)

Now in reference of (28), (29) and (30) we get from (2F)

Rᵃ (E, Y) Z
= R (E, Y) Z — g (Z, $E) $Y — g (Y, $Z) $E
—2g (Y, $E) $Z ‡ g (E, Z) 4 (Y) Ⓐ — 4 (E) g (Y, Z) Ⓐ
‡4 (E) 4 (Z) Y — 4 (Y) 4 (Z) E (3fi)

Consequently one can easily bring out the followings:

$$S^a(Y,Z) = S(Y,Z) ‡ 2g(Y,Z) — (fi ‡ n) 4(Y) 4(Z), \quad (32)$$
$$S^a(Y,Ⓐ) = 0, \quad (33)$$
$$S^a(Ⓐ,Z) = 0, \quad (34)$$
$$ⓟ^aY = ⓟY ‡ 2Y — (fi ‡ n) 4(Y) Ⓐ, \quad (3†)$$
$$ⓟ^aⒶ = 0, \quad (36)$$
$$v^a = v ‡ n — fi, \quad (3F)$$

$$R^a(E,Y)Ⓐ = 0, \quad (38)$$
$$R^a(Ⓐ,Y)Z = 0, \quad (39)$$
$$R^a(E,Ⓐ)Z = 0. \quad (40)$$

Thus we can state the followings:

Propositiom 2 Get M be an n-dsmenssona1 Sasahsan mansfo1d admsttsng Xamhoso4 GONNEGTSon ð^a, fhen
(s) fhe GUvsatuve tensov R^a of ð^a ss gssen b4 (hf),
(ss) fhe ÆSGGS tensov S^a of ð^a ss gssen b4 (hW),
(sss) fhe SGa1av GUvsatuve v^a of ð^a ss gssen b4 (hU),
(ss) fhe ÆSGGS tensov S^a of ð^a ss s4mmetvsG.

Now if we suppose that the Sasakian manifold is Ricci flat with respect to the Zamkovoy connection. Then from (32) we get

$$S(Y,Z) = —2g(Y,Z) ‡ (fi ‡ n) 4(Y) 4(Z).$$

This leads to the following

Theorem 3 If the mansfo1d M^n ss ÆSGGS fiat msth vespeGT to the Xamhoso4 GONNEGTSon sf and on14 sf M^n ss an 4-Esnstesn mansfo1d.

4 Comformally flat Sasakiam mamifold with respect to the commectiom ð^a

In view of (†), (3fi), (32) and (36), the conformal curvature tensor C^a with respect to the Zamkovoy connection ð^a on a Sasakian manifold M of dimension (n > 2) takes the form

$$\begin{aligned}
&C^a(E,Y)Z \\
&= C(E,Y)Z — g(Z,\$E)\$Y — g(Y,\$Z)\$E \\
&\quad —2g(Y,\$E)\$Z ‡ g(E,Z)4(Y)Ⓐ — 4(E)g(Y,Z)Ⓐ ‡ 4(E)4(Z)Y — 4(Y)4(Z)E \\
&\quad ‡ \frac{fi}{n—2}[—4g(Y,Z)E ‡ 4g(E,Z)Y ‡ (fi ‡ n)4(Y)4(Z)E — (fi ‡ n)4(E)4(Z)Y \\
&\qquad ‡ (fi ‡ n)4(E)g(Y,Z)Ⓐ — (fi ‡ n)4(Y)g(E,Z)Ⓐ] \qquad (4fi)
\end{aligned}$$

A conformal curvature tensor C is said to be flat with respect to Levi-Civita connection ð if it vanishes identically (that is, C = 0). A conformally flat Sasakian manifold with respect to the semi-symmetric metric connection has been studied in [2]. Assume that, M is conformally flat Sasakian manifold with respect to the connection $ð^a$. That is, $C^a = 0$. Then from (4fi), we have

R (E, Y) Z
= g (Z, $E) $Y ‡ g (Y, $Z) $E
‡2g (Y, $E) $Z — g (E, Z) 4 (Y) Ⓐ ‡ 4 (E) g (Y, Z) Ⓐ
—4 (E) 4 (Z) Y ‡ 4 (Y) 4 (Z) E
‡ fi/(n — 2) [S (Y, Z) E ‡ 2g (Y, Z) E — (fi ‡ n) 4 (Y) 4 (Z) E
—S (E, Z) Y — 2g (E, Z) Y ‡ (fi ‡ n) 4 (E) 4 (Z) Y
‡g (Y, Z) ⓟE ‡ 2g (Y, Z) E ‡ (fi ‡ n) 4 (E) g (Y, Z) Ⓐ
—g (E, Z) ⓟY — 2g (E, Z) Y ‡ (fi ‡ n) 4 (Y) g (E, Z) Ⓐ] (42)

Taking inner product with W in (42)

R (E, Y, Z, W) = g (Z, $E) g ($Y, W) ‡ g (Y, $Z) g ($E, W)
‡2g (Y, $E) g ($Z, W) — g (E, Z) 4 (Y) 4 (W) ‡ g (Y, Z) 4 (E) 4 (W)
—4 (E) 4 (Z) g (Y, W) ‡ 4 (Y) 4 (Z) g (E, W)
‡ fi/(n — 2) [S (Y, Z) g (E, W) ‡ 2g (Y, Z) g (E, W) — (fi ‡ n) 4 (Y) 4 (Z) g (E, W)
—S (E, Z) g (Y, W) — 2g (E, Z) g (Y, W) ‡ (fi ‡ n) 4 (E) 4 (Z) g (Y, W)
‡g (Y, Z) g (ⓟE, W) ‡ 2g (Y, Z) g (E, W) ‡ (fi ‡ n) 4 (E) g (Y, Z) g (Ⓐ, W)
—g (E, Z) g (ⓟY, W) — 2g (E, Z) g (Y, W) ‡ (fi ‡ n) 4 (Y) g (E, Z) g (Ⓐ, W)] (43)

Setting E = W = Ⓐ in (43)

R (Ⓐ, Y, Z, Ⓐ) = g (Z, $Ⓐ) g ($Y, Ⓐ) ‡ g (Y, $Z) g ($Ⓐ, Ⓐ)
‡2g (Y, $Ⓐ) g ($Z, Ⓐ) — g (Ⓐ, Z) 4 (Y) 4 (Ⓐ) ‡ g (Y, Z) 4 (Ⓐ) 4 (Ⓐ)
—4 (Ⓐ) 4 (Z) g (Y, Ⓐ) ‡ 4 (Y) 4 (Z) g (Ⓐ, Ⓐ)
‡ fi/(n — 2) [S (Y, Z) g (Ⓐ, Ⓐ) ‡ 2g (Y, Z) g (Ⓐ, Ⓐ) — (fi ‡ n) 4 (Y) 4 (Z) g (Ⓐ, Ⓐ)
—S (Ⓐ, Z) g (Y, Ⓐ) — 2g (Ⓐ, Z) g (Y, Ⓐ) ‡ (fi ‡ n) 4 (Ⓐ) 4 (Z) g (Y, Ⓐ)
‡g (Y, Z) g (ⓟⒶ, Ⓐ) ‡ 2g (Y, Z) g (Ⓐ, Ⓐ) ‡ (fi ‡ n) 4 (Ⓐ) g (Y, Z) g (Ⓐ, Ⓐ)
—g (Ⓐ, Z) g (ⓟY, Ⓐ) — 2g (Ⓐ, Z) g (Y, Ⓐ) ‡ (fi ‡ n) 4 (Y) g (Ⓐ, Z) g (Ⓐ, Ⓐ)] (44) By

the help of (fi4),(fi†), we have

S (Y, Z) = —2g (Y, Z) ‡ (n ‡ fi) 4 (Z) 4 (Y). (4†)

Thus, we can state the following:

Theorem 4 A GONFOVma114 fiat Sasahsan mansfo1d M (n > 2) admsttsng Xamhoso4 GONNEGTSon ð^a ss an 4-Esnstesn mansfo1d.

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5 Locally comformally $-symmetric Sasakiam mam-ifolds with respect to commectiom ð^a

In fi9FF, Takahashi[fi2] first studied the concept of locally $-symmetry on a Sasakian manifold. In this section we consider a locally conformally $-symmetric Sasakian manifolds with respect to the connection ð^a.

Defimitiom 5 A Sasahsan mansfo1d M ss sasd to be 1oGa114 GONFOVma114 $-s4mmetvsG msth vespeGT to the Xamhoso4 GONNEGTSON ð^a sf the GONFOVma1 GUVsatuve tensov C^a msth vespeGT to the GONNEGTSON ð^a satssfles

$$\$^2 (ð^a_V C^a)(E, Y) Z = 0, \quad (46)$$

mheve E, Y, Z and W ave hovssonta1 seGTOV fle1ds on M ,s.e E, Y, Z and W ave ovthonovma1 to Ⓐ on the sasahsan mansfo1d M.

In view of (22) we have that

$$(ð^a_V C^a)(E, Y) Z = (ð_V C^a)(E, Y) Z ‡ g(W, \$C^a (E, Y) Z) Ⓐ ‡ 4 (C^a (E, Y) Z) \$W$$
$$‡4 (W) \$C^a (E, Y) Z. \quad (4F)$$

Now, Differentiating (4fi) in the direction of W , we obtain

$$(ð_V C^a)(E, Y) Z = (ð_V C)(E, Y) Z. \quad (48)$$

Using (6), (fi4), (fi†) and (4fi)

$$4 (C^a (E, Y) Z) \$W$$
$$= —\frac{fi}{n — 2} [S (Y, Z) 4 (E) — S (E, Z) 4 (Y) ‡ 4g (Y, Z) 4 (E) — 4g (E, Z) 4 (Y)$$
$$— (fi ‡ n) 4 (E) g (Y, Z) ‡ (fi ‡ n) 4 (Y) g (E, Z)]\$W \quad (49)$$

By the help of (48) and (49), we get obtain from (4F)

$$(ð^a_V C^a)(E, Y) Z = (ð_V C)(E, Y) Z ‡ g (W, \$C^a (E, Y) Z) Ⓐ$$
$$—\frac{fi}{n— 2}[S (Y, Z) 4 (E) — S (E, Z) 4 (Y) ‡ 4g (Y, Z) 4 (E) — 4g (E, Z) 4 (Y) — (fi ‡ n) 4 (E) g (Y, Z)$$
$$‡ (fi ‡ n) 4 (Y) g (E, Z)]\$W ‡ 4 (W) \$C^a (E, Y) Z. \quad (†0)$$

Applying $\$^2$ on both sides of (†0) and using (6), we obtain

$$\$^2 (ð^a_V C^a)(E, Y) Z$$
$$= \$^2 (ð_V C)(E, Y) Z$$
$$‡ \frac{fi}{n — fi} [S (Y, Z) 4 (E) — S (E, Z) 4 (Y) ‡ 2g (Y, Z) 4 (E) — 2g (E, Z) 4 (Y)] \$W$$
$$‡4 (W) \$C^a (E, Y) Z. \quad (†fi)$$

Now, if E, Y, W are horizantal vector fields,i.e if orthogonal to Ⓐ, then ,4 (E) = 4 (Y) = 4 (W) = 0 and the above equation reduces to

$$\$^2 (ð^a_\vee C^a) (E, Y) Z = \$^2 (ð_\vee C) (E, Y) Z. \qquad (†2)$$

Which shows that, the manifold M is locally conformally \$-symmetric with respect to the connection $ð^a$ if and only if it is so with respect to the connection ð.

Thus we state the following:

Theorem 6 A Sasahsan mansfo1d M (n > 3), ss 1oGa114 GONFOVma114 \$-s4mmetvsG msth vespeGT to Xamhoso4 GONNEGTSon $ð^a$ sf and on14 sf st ss so msth vespeGT to the Gess-Gsssta GONNEGTSON ð.

6 Sasakiam mamifold admittimg Eamkovoy com-mectiom $ð^a$ satisfyimg C (Ⓐ, U) ∘ R = 0

In this section consider a Sasakian manifold M satisfying the condition

$$C^a (Ⓐ, U) \circ R^a (W, Z) V = 0. \qquad (†3)$$

Then we have

$$C^a (Ⓐ, U) R^a (W, Z) V - R^a (C^a (Ⓐ, U) W, Z) V - R^a (W, C^a (Ⓐ, U) Z) V - R^a (W, Z) C^a (Ⓐ, U) V = 0. \qquad (†4)$$

Replacing W by Ⓐ in (†4), we get

$$C^a (Ⓐ, U) R^a (Ⓐ, Z) V = R^a (C^a (Ⓐ, U) Ⓐ, Z) V ‡ R^a (Ⓐ, C^a (Ⓐ, U) Z) V ‡ R^a (Ⓐ, Z) C^a (Ⓐ, U) V = 0. \qquad (††)$$

Using (38), (39) and (40) in (††)

$$R^a (C^a (Ⓐ, U) Ⓐ, Z) V = 0. \qquad (†6)$$

By the help of (3fi) and (4fi) in (†6) we get

$$0 = R^a (ⓟU, Z) V ‡ 2R^a (U, Z) V. \qquad (†F)$$

Taking inner product with Y in the above equation we obtain

$$R^a (ⓟU, Z, V, Y) ‡ 2R^a (U, Z, V, Y) = 0. \qquad (†8)$$

Let $(e_s\}$ (fi ≤ s ≤ n) be an orthonormal basis of the tangent space at any point of the manifold M. Then putting U = Y = e_s in the equation (†8) and taking summation over s , fi ≤ s ≤ n, we get

$$\begin{aligned} & S^2 (Z, V) \\ = & -3S (\$V, \$Z) - 2S (Z, V) ‡ \dot{n}^2 - n ‡ 4 \sum 4 (V) 4 (Z) \\ & ‡ (n - †) g (V, Z) . \end{aligned} \qquad (†9)$$

This leads the following theorem:

Theorem Y In an n-dsmenssona1 (n > 3) Sasahsan mansfo1d M admsttsng Xamhoso4 GONNEGTSon ð^a, sf the GONDSTSON C (Ⓐ, U) ○R = 0 ho1ds on M, then the equatson (†9) ss satssfled on M.

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