DECOMPOSITION OF $(\acute{\alpha}_m, \acute{\lambda})$-CONTINUITY

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Abstract: In this paper, we introduce and study the notions of $\acute{\gamma}_m(\widetilde{\mu})$-open sets, where $\acute{\gamma} \in \{\widetilde{\mu}, \alpha, \sigma, \pi, \beta\}$. Also we obtain decomposition $(\acute{\alpha}_m, \acute{\lambda})$-continuity.

1 Introduction

In the year 2002, Csaszar [1] introduced very usefull notions of generalized topology $(G.T.)$ and generalized continuity. A subset L of a space $(Z, \widetilde{\mu})$ is $\widetilde{\mu}$-σ-*ópen* [2] (resp. $\widetilde{\mu}$-α-*ópen* [2], $\widetilde{\mu}$-π-*ópen*, $\widetilde{\mu}$-β-*ópen [2]*), if $L \subset c_{\widetilde{\mu}} i_{\widetilde{\mu}}(L)$ (resp. $L \subset i_{\widetilde{\mu}} c_{\widetilde{\mu}} i_{\widetilde{\mu}}(L)$, $L \subset i_{\widetilde{\mu}} c_{\widetilde{\mu}}(L)$, $L \subset c_{\widetilde{\mu}} i_{\widetilde{\mu}} c_{\widetilde{\mu}}(L)$). Let us denote by $\sigma(\widetilde{\mu})$ (resp. $\pi(\widetilde{\mu})$, $\alpha(\widetilde{\mu})$, $\beta(\widetilde{\mu})$) the class of all $\widetilde{\mu}$-σ-*ópen* sets (resp. $\widetilde{\mu}$-π-*ópen* sets, $\widetilde{\mu}$-α-*ópen* sets, $\widetilde{\mu}$-β-*ópen* sets). Let Z be a non empty set and $\widetilde{\mu}$ be a generalized topology and $\widetilde{m}$ a minimal structure on Z. A triple $(Z, \widetilde{\mu}, \widetilde{m})$ is called a *generalized topology and minimal structure space [6] (briefly G.T.M.S. space)*. Let $(Z, \widetilde{\mu}, \widetilde{m})$ be a $G.T.M.S.$ and L a subset of Z. The closure and interior of L in $\widetilde{m}$ are denoted by $c_{\widetilde{m}}(L)$ and $i_{\widetilde{m}}(L)$, respectively. In this paper the minimal structure $\widetilde{m}$ is closed under arbitrary union.

2 Preliminaries

Definition 2.1. *[6] Let $(Z, \widetilde{\mu}, \widetilde{m})$ be a G.T.M.S. space. A subset L of Z is said to be $\widetilde{\mu}\widetilde{m}$-closed if $c_{\widetilde{\mu}} c_{\widetilde{m}}(L) = L$. A subset L of Z is said to be $\widetilde{m}\widetilde{\mu}$-closed if*

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$c_{\widetilde{m}}c_{\widetilde{\mu}}(L) = L$.

Lemma 2.2. *[6] Let $(Z, \widetilde{\mu}, \widetilde{m})$ be a G.T.M.S. space and $L \subset Z$. Then*

1. *L is $\widetilde{\mu}\widetilde{m}$-closed if and only if $c_{\widetilde{m}}(L) = L$ and $c_{\widetilde{\mu}}(L) = L$.*
2. *L is $\widetilde{m}\widetilde{\mu}$-closed if and only if $c_{\widetilde{m}}(L) = L$ and $c_{\widetilde{\mu}}(L) = L$.*

Definition 2.3. *[6] Let $(Z, \widetilde{\mu}, \widetilde{m})$ be a G.T.M.S. A subset L of Z is said to be s-closed if $c_{\widetilde{\mu}}(L) = c_{\widetilde{m}}(L)$. A subset L of Z is said to be c-closed if $c_{\widetilde{\mu}}c_{\widetilde{m}}(L) = c_{\widetilde{m}}c_{\widetilde{\mu}}(L)$. The complement of s-closed (resp. c-closed) set is called s-open (resp. c-open) set.*

Proposition 2.4. *Let $(Z, \widetilde{\mu}, \widetilde{m})$ be G.T.M.S, where $\widetilde{m}$ is closed under union. Then*

1. *$L \subset M \subset Z$ implies $i_{\widetilde{m}}(L) \subset i_{\widetilde{m}}(M)$,*
2. *$i_{\widetilde{m}}(L) \subset L$,*
3. *$i_{\widetilde{m}}(i_{\widetilde{m}}(L)) = i_{\widetilde{m}}(L)$,*
4. *$i_{\widetilde{m}}(L) = L$, if $L \in \widetilde{m}$,*
5. *L is $\widetilde{m}$-closed if and only if $c_{\widetilde{m}}(L) = L$.*

Proof. (1) and (2) are evident.

3. By (2) $i_{\widetilde{m}}i_{\widetilde{m}}(L) \subset i_{\widetilde{m}}(L)$. On the other hand if $M \in \widetilde{m}$ and $M \subset L$ then $M \subset i_{\widetilde{m}}(L)$ by definition so that $M \subset i_{\widetilde{m}}i_{\widetilde{m}}(L)$ by definition again, consequently $i_{\widetilde{m}}(L) \subset i_{\widetilde{m}}i_{\widetilde{m}}(L)$ and $i_{\widetilde{m}}(L) = i_{\widetilde{m}}i_{\widetilde{m}}A$.

4. If $L \in \widetilde{m}$, then $i_{\widetilde{m}}(L) = \bigcup\{F : F \subset m, F \subseteq L\} = L$. Conversely, since $\widetilde{m}$ is closed under arbitrary union, then $i_{\widetilde{m}}(L) \in \widetilde{m}$. It follows that $L \in \widetilde{m}$.

5. If L is $\widetilde{m}$-closed set, then $Z - L \in \widetilde{m}$. By definition of $i_{\widetilde{m}}$, $i_{\widetilde{m}}(Z - L) = Z - L$, $i_{\widetilde{m}}(Z - L) = Z - c_{\widetilde{m}}(L)$. In consequence $c_{\widetilde{m}}(L) = L$.

3 $\acute{\gamma}_m(\widetilde{\mu})$-open sets

Definition 3.1. *A subset L of G.T.M.S. $(Z, \widetilde{\mu}, \widetilde{m})$ is said to be*

1. $\widetilde{m}_{\widetilde{\mu}}$-$\alpha$-*open, if* $L \subset i_{\widetilde{m}} c_{\widetilde{\mu}} i_{\widetilde{m}}(L)$
2. $\widetilde{m}_{\widetilde{\mu}}$-$\sigma$-*open, if* $L \subset c_{\widetilde{\mu}} i_{\widetilde{m}}(L)$
3. $\widetilde{m}_{\widetilde{\mu}}$-$\pi$-*open, if* $L \subset i_{\widetilde{m}} c_{\widetilde{\mu}}(L)$
4. $\widetilde{m}_{\widetilde{\mu}}$-$\beta$-*open, if* $L \subset c_{\widetilde{\mu}} i_{\widetilde{m}} c_{\widetilde{\mu}}(L)$.

Let $\widetilde{\mu}$ be a generalized topology, $\widetilde{m}$ be a minimal structure on Z and $\widetilde{m}$ is closed under union. The element of $(Z, \widetilde{\mu}, \widetilde{m})$ are called $\widetilde{m}_{\widetilde{\mu}}$-open sets. Let us denote $\widetilde{\mu}_m(\widetilde{\mu})$ the class of all $\widetilde{m}_{\widetilde{\mu}}$-open sets, $\alpha_{\widetilde{m}}(\widetilde{\mu})$ the class of all $\widetilde{m}_{\widetilde{\mu}}$-$\alpha$-open sets by $\sigma_{\widetilde{m}}(\widetilde{\mu})$ that of all $\widetilde{m}_{\widetilde{\mu}}$-$\sigma$-open by $\pi_{\widetilde{m}}(\widetilde{\mu})$ that of all $\widetilde{m}_{\widetilde{\mu}}$-$\pi$-open sets, by $\beta_{\widetilde{m}}(\widetilde{\mu})$ that of all $\widetilde{m}_{\widetilde{\mu}}$-$\beta$-open sets.

Remark 3.2. *The notions of $\widetilde{\mu}$-α-open (resp. $\widetilde{\mu}$-σ-open, $\widetilde{\mu}$-π-open, $\widetilde{\mu}$-β-open) and $\widetilde{m}_{\widetilde{\mu}}$-$\alpha$-open (resp. $\widetilde{m}_{\widetilde{\mu}}$-$\sigma$-open, $\widetilde{m}_{\widetilde{\mu}}$-$\pi$-open, $\widetilde{m}_{\widetilde{\mu}}$-$\beta$-open) sets are independent.*

Example 3.3. *Let $Z = \mathbb{R}$ be the set of all real numbers, $\widetilde{\mu} = \{\emptyset, \mathbb{Q}\}$ and $\widetilde{m} = \{\emptyset, \mathbb{Q}, \mathbb{R}\}$, where $\mathbb{Q}$ is set of all rational numbers. Then $L = \{\mathbb{Q} \cup \sqrt{2}\}$ is $\widetilde{m}_{\widetilde{\mu}}$-$\alpha$-open but not $\widetilde{\mu}$-α-open.*

Example 3.4. *Let $Z = \mathbb{R}$ be the set of all real numbers, $\widetilde{\mu} = \{\emptyset, \mathbb{Q}\}$ and $\widetilde{m} = \{\emptyset, \mathbb{R} - \mathbb{Q}, \mathbb{R}\}$, where $\mathbb{Q}$ is set of all rational numbers and $\mathbb{R} - \mathbb{Q}$ is set of all irrational numbers. Then $L = \mathbb{Q}$ is $\widetilde{\mu}$-α-open but not $\widetilde{m}_{\widetilde{\mu}}$-$\alpha$-open and $B = \mathbb{R} - \mathbb{Q}$ is $\widetilde{m}_{\widetilde{\mu}}$-$\pi$-open but not $\widetilde{\mu}$-π-open.*

Example 3.5. *Let $Z = \mathbb{R}$ be the set of all real numbers, $\widetilde{\mu} = \{\emptyset, \mathbb{R} - \mathbb{Q}\}$ and $\widetilde{m} = \{\emptyset, \mathbb{Q} \cup \sqrt{2}, \mathbb{R}\}$, where $\mathbb{Q}$ is set of all rational numbers and $\mathbb{R} - \mathbb{Q}$ is set of all irrational numbers. Then $L = \mathbb{Q} \cup \sqrt{2}$ is $\widetilde{m}_{\widetilde{\mu}}$-$\sigma$-open but not $\widetilde{\mu}$-σ-open.*

Example 3.6. *Let* $Z = \mathbb{R}$ *be the set of all real numbers,* $\widetilde{\mu} = \{\emptyset, \mathbb{R} - \mathbb{Q}, \mathbb{Q}, \mathbb{R}\}$ *and* $\widetilde{m} = \{\emptyset, \mathbb{R} - \mathbb{Q}, \mathbb{R}\}$, *where* $\mathbb{Q}$ *is set of all rational numbers and* $\mathbb{R} - \mathbb{Q}$ *is set of all irrational numbers. Then* $L = \mathbb{Q}$ *is* $\widetilde{\mu}$ *-* π *-open (resp.* $\widetilde{\mu}$ *-* β *-open) but not* $\widetilde{m}_{\widetilde{\mu}}$ *-* π *-open (resp.* $\widetilde{m}_{\widetilde{\mu}}$ *-* β *-open).*

Example 3.7. *Let* $Z = \mathbb{R}$ *be the set of all real numbers,* $\widetilde{\mu} = \{\emptyset, \mathbb{R} - \mathbb{Q}\}$ *and* $\widetilde{m} = \{\emptyset, \mathbb{Q}, \mathbb{R}\}$, *where* $\mathbb{Q}$ *is set of all rational numbers and* $\mathbb{R} - \mathbb{Q}$ *is set of all irrational numbers. Then* $L = \mathbb{R} - \mathbb{Q}$ *is* $\widetilde{\mu}$ *-* σ *- open but not* $\widetilde{m}_{\widetilde{\mu}}$ *-* σ *-open.*

Example 3.8. *Let* $Z = \mathbb{R}$ *be the set of all real numbers,* $\widetilde{\mu} = \{\emptyset, \mathbb{R} - \mathbb{Q} - \sqrt{2}, \mathbb{R} - \mathbb{Q}\}$ *and* $\widetilde{m} = \{\emptyset, \mathbb{Q} \cup \sqrt{2}, \mathbb{R}\}$, *where* $\mathbb{Q}$ *is set of all rational numbers and* $\mathbb{R} - \mathbb{Q}$ *is set of all irrational numbers. Then* $L = \mathbb{Q} \cup \sqrt{2}$ *is* $\widetilde{m}_{\widetilde{\mu}}$ *-* β *-open but not* $\widetilde{\mu}$ *-* β *-open.*

Theorem 3.9. *For a generalized topology and minimal structure space* $(Z, \widetilde{\mu}, \widetilde{m})$, *we have*

1. $\acute{\alpha}_m(\widetilde{\mu}) \subset \sigma_m(\widetilde{\mu}) \subset \beta_m(\widetilde{\mu})$
2. $\acute{\alpha}_m(\widetilde{\mu}) \subset \pi_{\widetilde{m}}(\widetilde{\mu}) \subset \beta_m(\widetilde{\mu})$

Proof.

1. Clearly, $i_{\widetilde{m}}(c_{\widetilde{\mu}}(i_{\widetilde{m}}(L))) \subset c_{\widetilde{\mu}}(i_{\widetilde{m}}(L)) \subset c_{\widetilde{\mu}}(i_{\widetilde{m}}(c_{\widetilde{\mu}}(L)))$
2. Clearly, $i_{\widetilde{m}}(c_{\widetilde{\mu}}(i_{\widetilde{m}}(L))) \subset i_{\widetilde{m}}c_{\widetilde{\mu}}(L)) \subset c_{\widetilde{\mu}}(i_{\widetilde{m}}(c_{\widetilde{\mu}}(L)))$.

Remark 3.10. *The following Examples shows that the converese of Theorem* 3.9 *need not be true.*

Example 3.11. *Let* $Z = \{\varsigma_1, \varsigma_2, \varsigma_3, \varsigma_4, \varsigma_5\}$, $\widetilde{\mu} = \{\emptyset, \{\varsigma_1\}, \{\varsigma_2\}, \{\varsigma_3\}, \{\varsigma_1, \varsigma_2\}, \{\varsigma_1, \varsigma_3\}, \{\varsigma_2, \varsigma_3\}, \{\varsigma_1, \varsigma_2, \varsigma_3\}, \{\varsigma_1, \varsigma_3, \varsigma_4\}, \{\varsigma_2, \varsigma_3, \varsigma_4\}, \{\varsigma_1, \varsigma_2, \varsigma_4\}, \{\varsigma_1, \varsigma_2, \varsigma_3, \varsigma_4\}$ *and* $\widetilde{m} = \{\emptyset, \{\varsigma_1, \varsigma_2\}, \{\varsigma_1, \varsigma_3\}, \{\varsigma_1, \varsigma_2, \varsigma_3\}, \{\varsigma_2, \varsigma_3, \varsigma_4, \varsigma_5\}, Z\}$. $L = \{\varsigma_1, \varsigma_4\}$ *is* $\widetilde{m}_{\widetilde{\mu}}$ *-* σ *-open but not* $\widetilde{m}_{\widetilde{\mu}}$ *-* α *-open.*

Example 3.12. *Let* $Z = \mathbb{R}$ *be the set of all real numbers,* $\widetilde{\mu} = \{\emptyset, \mathbb{R} - \mathbb{Q} - \sqrt{2}\}$ *and* $\widetilde{m} = \{\emptyset, \mathbb{Q}, \mathbb{R}\}$, *where* $\mathbb{Q}$ *is set of all rational numbers and* $\mathbb{R} - \mathbb{Q}$ *is set of all irrational numbers. Then* $L = \mathbb{R} - \mathbb{Q}$ *is* $\widetilde{m}_{\widetilde{\mu}}$*-*$\pi$*-open but not* $\widetilde{m}_{\widetilde{\mu}}$*-*$\alpha$*-open.*

Example 3.13. *Let* $Z = \mathbb{R}$ *be the set of all real numbers,* $\widetilde{\mu} = \{\emptyset, \mathbb{R} - \mathbb{Q}\}$ *and* $\widetilde{m} = \{\emptyset, \mathbb{R} - \mathbb{Q}, \mathbb{R}\}$, *where* $\mathbb{Q}$ *is set of all rational numbers and* $\mathbb{R} - \mathbb{Q}$ *is set of all irrational numbers. Then* $L = \mathbb{Q}$ *is* $\widetilde{m}_{\widetilde{\mu}}$*-*$\beta$*-open but not* $\widetilde{m}_{\widetilde{\mu}}$*-*$\pi$*-open and* $B = \mathbb{Q} \cup \sqrt{2}$ *is* $\widetilde{m}_{\widetilde{\mu}}$*-*$\beta$*-open but not* $\widetilde{m}_{\widetilde{\mu}}$*-*$\sigma$*-open(resp.* $\widetilde{m}_{\widetilde{\mu}}$*-*$\alpha$*-open).*

Theorem 3.14. *Let* $(Z, \widetilde{\mu}, \widetilde{m})$ *be a G.T.M.S. space and* $L \subset X$. *Then the following are equvalent.*

1. L *is* $\widetilde{m}_{\widetilde{\mu}}$*-*$\alpha$*-open*
2. L *is* $\widetilde{m}_{\widetilde{\mu}}$*-*$\sigma$*-open and* $\widetilde{m}_{\widetilde{\mu}}$*-*$\pi$*-open.*

Proof. $(1) \Rightarrow (2)$. Let L is $\widetilde{m}_{\widetilde{\mu}}$-$\alpha$-open. Then by Theorem 3.9, L is $\widetilde{m}_{\widetilde{\mu}}$-$\sigma$-open and $\widetilde{m}_{\widetilde{\mu}}$-$\pi$-open.

$(2) \Rightarrow (1)$. Let L is $\widetilde{m}_{\widetilde{\mu}}$-$\sigma$-open and $\widetilde{m}_{\widetilde{\mu}}$-$\pi$-open. Then $L \subset i_{\widetilde{m}}c_{\widetilde{\mu}}(L) \subset i_{\widetilde{m}}c_{\widetilde{\mu}}c_{\widetilde{\mu}}i_{\widetilde{m}}(L) = i_{\widetilde{m}}c_{\widetilde{\mu}}i_{\widetilde{m}}(L)$. Hence L is $\widetilde{m}_{\widetilde{\mu}}$-$\alpha$-open.

Remark 3.15. *The notions of* $\widetilde{m}_{\widetilde{\mu}}$*-*$\sigma$*-open and* $\widetilde{m}_{\widetilde{\mu}}$*-*$\pi$*-open are independent.*

Example 3.16. *Let* $Z = \{\varsigma_1, \varsigma_2, \varsigma_3, \varsigma_4, \varsigma_4\}$, $\widetilde{\mu} = \{\emptyset, \{\varsigma_1\}, \{\varsigma_2\}, \{\varsigma_3\}, \{\varsigma_1, \varsigma_2\}, \{\varsigma_1, \varsigma_3\}, \{\varsigma_2, \varsigma_3\}, \{\varsigma_1, \varsigma_2, \varsigma_3\}, \{\varsigma_1, \varsigma_3, \varsigma_4\}, \{\varsigma_2, \varsigma_3, \varsigma_4\}, \{\varsigma_1, \varsigma_2, \varsigma_4\}, \{\varsigma_1, \varsigma_2, \varsigma_3, \varsigma_4\}$ *and* $\widetilde{m} = \{\emptyset, \{\varsigma_1, \varsigma_2\}, \{\varsigma_1, \varsigma_3\}, \{\varsigma_1, \varsigma_2, \varsigma_3\}, \{\varsigma_2, \varsigma_3, \varsigma_4, \varsigma_5\}, Z\}$. $L = \{\varsigma_1, \varsigma_4\}$ *is* $\widetilde{m}_{\widetilde{\mu}}$*-*$\sigma$*-open but not* $\widetilde{m}_{\widetilde{\mu}}$*-*$\pi$*-open.*

Example 3.17. *Let* $Z = \mathbb{R}$ *be the set of all real numbers,* $\widetilde{\mu} = \{\emptyset, \mathbb{R} - \mathbb{Q}\}$ *and* $\widetilde{m} = \{\emptyset, \mathbb{R} - \mathbb{Q}, \mathbb{R}\}$, *where* $\mathbb{Q}$ *is set of all rational numbers and* $\mathbb{R} - \mathbb{Q}$ *is set of all irrational numbers. Then* $L = \mathbb{Q} \cup \sqrt{2}$ *is* $\widetilde{m}_{\widetilde{\mu}}$*-*$\pi$*-open but not* $\widetilde{m}_{\widetilde{\mu}}$*-*$\sigma$*-open.*

Theorem 3.18. *If* $L \subset Z$ *is* $\widetilde{m}_{\widetilde{\mu}}$*-*$\sigma$*-open if and only if* $c_{\widetilde{\mu}}(L) = c_{\widetilde{\mu}}i_{\widetilde{m}}(L)$.

Proof. Let a subset L of Z is $\widetilde{m}_{\widetilde{\mu}}$-$\sigma$-open. Then $L \subset c_{\widetilde{\mu}}i_{\widetilde{m}}(L)$. Now $c_{\widetilde{\mu}}(L) \subset c_{\widetilde{\mu}}c_{\widetilde{\mu}}i_{\widetilde{m}}(L) = c_{\widetilde{\mu}}i_{\widetilde{m}}(L)$. Always $c_{\widetilde{\mu}}i_{\widetilde{m}}(L) \subset c_{\widetilde{\mu}}(L)$. Hence $c_{\widetilde{\mu}}(L) = c_{\widetilde{\mu}}i_{\widetilde{m}}(L)$.

Conversely, let $c_{\widetilde{\mu}}(L) = c_{\widetilde{\mu}} i_{\widetilde{m}}(L)$. Then $L \subset c_{\widetilde{\mu}}(L) = c_{\widetilde{\mu}} i_{\widetilde{m}}(L)$. Hence L is $\widetilde{m}_{\widetilde{\mu}}$-$\sigma$-open.

Theorem 3.19. *If $L \subset Z$ is both $\widetilde{\mu}$-closed and $\widetilde{m}_{\widetilde{\mu}}$-$\pi$-open, then it is $\widetilde{m}_{\widetilde{\mu}}$-open.*

Proof. Let a subset L of Z is both $\widetilde{\mu}$-closed and $\widetilde{m}_{\widetilde{\mu}}$-$\pi$-open. Then $L \subset i_{\widetilde{m}} c_{\widetilde{\mu}}(L) = i_{\widetilde{m}}(L)$. Hence L is $\widetilde{m}_{\widetilde{\mu}}$-open.

Theorem 3.20. *If $L \subset Z$ is both $\widetilde{\mu}$-closed and $\widetilde{m}_{\widetilde{\mu}}$-$\beta$-open, then it is $\widetilde{m}_{\widetilde{\mu}}$-$\sigma$-open.*

Proof. Let a subset L of Z is both $\widetilde{\mu}$-closed and $\widetilde{m}_{\widetilde{\mu}}$-$\beta$-open. Then $L \subset c_{\widetilde{\mu}} i_{\widetilde{m}} c_{\widetilde{\mu}}(L) = c_{\widetilde{\mu}} i_{\widetilde{m}}(L)$. Hence L is $\widetilde{m}_{\widetilde{\mu}}$-$\sigma$-open.

Theorem 3.21. *If $L \subset Z$ is both $\widetilde{\mu}_m$-closed and $\widetilde{m}_{\widetilde{\mu}}$-$\alpha$-open, then it is $\widetilde{m}_{\widetilde{\mu}}$-open.*

Proof. Let a subset L of Z is both $\widetilde{\mu}_m$-closed and $\widetilde{m}_{\widetilde{\mu}}$-$\alpha$-open. Then $L \subset i_{\widetilde{m}} c_{\widetilde{\mu}} i_{\widetilde{m}}(L) \subset i_{\widetilde{m}} c_{\widetilde{\mu}} c_{\widetilde{m}}(L) = i_{\widetilde{m}}(L)$. Hence L is $\widetilde{m}_{\widetilde{\mu}}$-open.

Theorem 3.22. *If $L \subset Z$ is both $\widetilde{\mu}_m$-closed and $\widetilde{m}_{\widetilde{\mu}}$-$\beta$-open, then it is $\widetilde{m}_{\widetilde{\mu}}$-$\sigma$-open.*

Proof. Let a subset L of Z is both $\widetilde{\mu}_m$-closed and $\widetilde{m}_{\widetilde{\mu}}$-$\beta$-open. Then $L \subset c_{\widetilde{\mu}} i_{\widetilde{m}} c_{\widetilde{\mu}}(L) = c_{\widetilde{\mu}} i_{\widetilde{m}}(L)$. Hence L is $\widetilde{m}_{\widetilde{\mu}}$-$\sigma$-open.

Theorem 3.23. *If $L \subset Z$ is both $\widetilde{\mu}_m$-closed and $\widetilde{m}_{\widetilde{\mu}}$-$\pi$-open, then it is $\widetilde{m}_{\widetilde{\mu}}$-open.*

Proof. Let a subset L of Z is both $\widetilde{\mu}_m$-closed and $\widetilde{m}_{\widetilde{\mu}}$-$\pi$-open. Then $L \subset i_{\widetilde{m}} c_{\widetilde{\mu}}(L) = i_{\widetilde{m}}(L)$. Hence L is $\widetilde{m}_{\widetilde{\mu}}$-open.

Theorem 3.24. *If $L \subset Z$ is both s-open and $\widetilde{m}_{\widetilde{\mu}}$-$\alpha$-open, then it is $\widetilde{\mu}$-σ-open.*

Proof. Let a subset L of Z is both s-open and $\widetilde{m}_{\widetilde{\mu}}$-$\alpha$-open. Then $L \subset c_{\widetilde{\mu}} i_{\widetilde{m}}(L) = c_{\widetilde{\mu}} i_{\widetilde{\mu}}(L)$. Hence L is $\widetilde{\mu}$-σ-open.

Theorem 3.25. *If $L \subset Z$ is both $\widetilde{m}_{\widetilde{\mu}}$-closed and $\widetilde{m}_{\widetilde{\mu}}$-$\pi$-open, then $L = i_{\widetilde{m}} c_{\widetilde{\mu}}(L)$.*

Proof. Let a subset L of Z is both $\widetilde{m}_{\widetilde{\mu}}$-closed and $\widetilde{m}_{\widetilde{\mu}}$-$\pi$-open. Then $L \subset i_{\widetilde{m}}c_{\widetilde{\mu}}(L)$ and $c_{\widetilde{m}}c_{\widetilde{\mu}}(L) = L$. Now $i_{\widetilde{m}}c_{\widetilde{\mu}}(L) \subset c_{\widetilde{m}}c_{\widetilde{\mu}}(L) = L$. Hence L is $\widetilde{m}_{\widetilde{\mu}}$-open.

Theorem 3.26. *If $L \subset Z$ is both $\widetilde{\mu}_m$-closed and $\widetilde{m}_{\widetilde{\mu}}$-$\sigma$-open, then $L = c_{\widetilde{\mu}}i_{\widetilde{m}}(L)$.*

Proof. Let a subset L of Z is both $\widetilde{\mu}_m$-closed and $\widetilde{m}_{\widetilde{\mu}}$-$\sigma$-open. Then $L \subset c_{\widetilde{\mu}}i_{\widetilde{m}}(L)$ and $c_{\widetilde{\mu}}c_{\widetilde{m}}(L) = L$. Now $c_{\widetilde{\mu}}i_{\widetilde{m}}(L) \subset c_{\widetilde{\mu}}c_{\widetilde{m}}(L) = L$. Hence $L = c_{\widetilde{\mu}}i_{\widetilde{m}}(L)$.

Theorem 3.27. *If $L \subset Z$ is both $\widetilde{\mu}$-σ-open and s-open, then it is $\widetilde{m}_{\widetilde{\mu}}$-$\sigma$-open.*

Proof. Let $L \subset Z$ is both $\widetilde{\mu}$-σ-open and s-open, then it is $\widetilde{m}_{\widetilde{\mu}}$-$\sigma$-open. Then $L \subset c_{\widetilde{\mu}}i_{\widetilde{\mu}}(L)$ and $i_{\widetilde{\mu}}(L) = i_{\widetilde{m}}(L)$. Now $L \subset c_{\widetilde{\mu}}i_{\widetilde{\mu}}(L) = c_{\widetilde{\mu}}i_{\widetilde{m}}(L)$. Hence L is $\widetilde{m}_{\widetilde{\mu}}$-$\sigma$-open.

4 Decomposition of $(\acute{\alpha}_m, \acute{\lambda})$-continuity

Definition 4.1. *A map $f : (Z, \widetilde{\mu}, \widetilde{m}) \to (Y, \acute{\lambda})$ is said to be $(\acute{\alpha}_m, \acute{\lambda})$-continuous (resp. $(\sigma_m, \acute{\lambda})$-continuous, $(\pi_{\widetilde{m}}, \acute{\lambda})$-continuous, $(\beta_m, \acute{\lambda})$-continuous), if for each $\acute{\lambda}$-open set U in $(Y, \acute{\lambda})$, $f^{-1}(U)$ is $\widetilde{m}_{\widetilde{\mu}}$-$\alpha$-open (resp. $\widetilde{m}_{\widetilde{\mu}}$-$\sigma$-open, $\widetilde{m}_{\widetilde{\mu}}$-$\pi$-open, $\widetilde{m}_{\widetilde{\mu}}$-$\beta$-open) set in $(Z, \widetilde{\mu}, \widetilde{m})$.*

Theorem 4.2. *Every $(\acute{\alpha}_m, \acute{\lambda})$-continuous is $(\sigma_m, \acute{\lambda})$-continuous but not conversely.*

Proof. The proof is follows from Theorem 3.9.

Example 4.3. *$Z = Y = \{\varsigma_1, \varsigma_2, \varsigma_3\}$, $\widetilde{\mu} = \{\emptyset, \{\varsigma_1\}\}$, $\widetilde{m} = \{\emptyset, \{\varsigma_1\}, Z\}$, and $\acute{\lambda} = \{\emptyset, \{\varsigma_2\}, \{\varsigma_2, \varsigma_3\}\}$. Then the identity function f is $(\sigma_m, \acute{\lambda})$-continuous but not $(\acute{\alpha}_m, \acute{\lambda})$-continuous.*

Theorem 4.4. *Every $(\acute{\alpha}_m, \acute{\lambda})$-continuous is $(\pi_{\widetilde{m}}, \acute{\lambda})$-continuous but not conversely.*

Proof. The proof is follows from Theorem 3.9.

Example 4.5. *Let $Z = \{\varsigma_1, \varsigma_2, \varsigma_3\}$, $\widetilde{\mu} = \{\emptyset, \{\varsigma_1, \varsigma_3\}, \{\varsigma_2, \varsigma_3\}, Z\}$, $\widetilde{m} = \{\emptyset, \{\varsigma_1\}, Z\}$ and $\acute{\lambda} = \{\emptyset, \{\varsigma_1\}, \{\varsigma_2\}, \{\varsigma_1, \varsigma_2\}\}$. Then the identity function is $(\pi_{\widetilde{m}}, \acute{\lambda})$-continuous but $(\acute{\alpha}_m, \acute{\lambda})$-continuous.*

Theorem 4.6. *Every* $(\acute{\alpha}_m, \acute{\lambda})$ *-continuous (resp.* $(\sigma_m, \acute{\lambda})$ *-continuous,* $(\pi_{\widetilde{m}}, \acute{\lambda})$ *-continuous) is* $(\beta_m, \acute{\lambda})$ *-continuous but not conversely.*

Proof. The proof is follows from Theorem 3.9.

Example 4.7. *Let* $Z = Y = \{\varsigma_1, \varsigma_2, \varsigma_3\}$, $\widetilde{\mu} = \{\emptyset, \{\varsigma_1\}, \{\varsigma_2\}, \{\varsigma_1, \varsigma_2\}\}$, $\widetilde{m} = \{\emptyset, \{\varsigma_1\}, Z\}$ *and* $\acute{\lambda} = \{\emptyset, \{\varsigma_1, \varsigma_2\}, \{\varsigma_1, \varsigma_3\}, Y\}$. *Then the identity function* f *is* $(\beta_m, \acute{\lambda})$ *-continuous but not* $(\sigma_m, \acute{\lambda})$ *-continuous (resp.* $(\pi_{\widetilde{m}}, \acute{\lambda})$ *-continuous,* $(\acute{\alpha}_m, \acute{\lambda})$ *-continuous).*

Theorem 4.8. *A function* $f : (Z, \widetilde{\mu}, \widetilde{m}) \longrightarrow (Y, \acute{\lambda})$ *is* $(\acute{\alpha}_m, \acute{\lambda})$ *-continuous if and only if it is both* $(\sigma_m, \acute{\lambda})$ *-continuous and* $(\pi_{\widetilde{m}}, \acute{\lambda})$ *-continuous.*

Proof. The proof is follows from Theorem 3.14.

Remark 4.9. *The notions of* $(\sigma_m, \acute{\lambda})$ *-continuous and* $(\pi_{\widetilde{m}}, \acute{\lambda})$ *-continuous are independent.*

Example 4.10. *Let* $Z = W = \{\varsigma_1, \varsigma_2, \varsigma_3, \varsigma_4, \varsigma_4\}$, $\widetilde{\mu} = \{\emptyset, \{\varsigma_1\}, \{\varsigma_2\}, \{\varsigma_3\}, \{\varsigma_1, \varsigma_2\}, \{\varsigma_1, \varsigma_3\}, \{\varsigma_2, \varsigma_3\}, \{\varsigma_1, \varsigma_2, \varsigma_3\}, \{\varsigma_1, \varsigma_3, \varsigma_4\}, \{\varsigma_2, \varsigma_3, \varsigma_4\}, \{\varsigma_1, \varsigma_2, \varsigma_4\}, \{\varsigma_1, \varsigma_2, \varsigma_3, \varsigma_4\},$ $\widetilde{m} = \{\emptyset, \{\varsigma_1, \varsigma_2\}, \{\varsigma_1, \varsigma_3\}, \{\varsigma_1, \varsigma_2, \varsigma_3\}, \{\varsigma_2, \varsigma_3, \varsigma_4, \varsigma_5\}, Z\}$ *and* $\acute{\lambda} = \{\emptyset, \{\varsigma_1\}, \{\varsigma_2\}, \{\varsigma_3\}, \{\varsigma_1, \varsigma_2\}, \{\varsigma_1, \varsigma_3\}, \{\varsigma_2, \varsigma_3\}, \{\varsigma_1, \varsigma_2, \varsigma_3\}, \{\varsigma_1, \varsigma_3, \varsigma_4\}, \{\varsigma_2, \varsigma_3, \varsigma_4\}, \{\varsigma_1, \varsigma_2, \varsigma_4\}, \{\varsigma_1, \varsigma_2, \varsigma_3, \varsigma_4\}$. *Then the identity function* f *is* $(\sigma_m, \acute{\lambda})$ *-continuous but not* $(\pi_{\widetilde{m}}, \acute{\lambda})$ *-continuous.*

Example 4.11. *Let* $Z = Y = \mathbb{R}$ *be the set of all real numbers,* $\widetilde{\mu} = \acute{\lambda} = \{\emptyset, \mathbb{R} - \mathbb{Q}\}$ *and* $\widetilde{m} = \{\emptyset, \mathbb{R} - \mathbb{Q}, \mathbb{R}\}$, *where* $\mathbb{Q}$ *is set of all rational numbers and* $\mathbb{R} - \mathbb{Q}$ *is set of all irrational numbers. Then the identity function* f *is* $(\pi_{\widetilde{m}}, \acute{\lambda})$ *-continuous but not* $(\sigma_m, \acute{\lambda})$ *-continuous.*

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