A study on MHD fluid flow in parallel surface channel

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Abstract- We discuss the steady MHD flow of an incompressible fluid in a parallel plate channel bounded on one side by a porous bed. The accurate solutions of the velocity in the fluid and the porous medium consists of steady state are analytically derived, its behavior computationally discussed with reference to the various governing parameters with the help of graphs. The shear stresses on the boundaries and the mass flux are also obtained analytically and their behavior is computationally discussed.

Keywords – ***MHD, Inclined, steady state, Hall current.***

I. INTRODUCTION

MHD fluid flows have a number of important applications such as MHD power generator, an electromagnetic flow meter or electromagnetic accelerators. The last device has been used in connection with a nuclear power reactor to pump liquid sodium as a coolant. Hence it is important to know the effects of couple stresses on the flow field in such devices. In conventional MHD flow has been the subject of many investigations. Most of these studies pertain to Newtonian fluids. Newtonian fluid theory has been introduced which takes care of presence of couple stresses in fluids. The couple stress fluid theory presents models for fluids whose microstructure is mechanically significant. The effect of very small microstructure in a fluid can be felt if the characteristic geometric dimension of the problem considered is of the same order of magnitude as the size of the microstructure. Dharmaiah and veerakrishna [1] explained mhd free convection flow through porous medium along a vertical wall with finite difference method. Veera Krishna and Dharmaiah [2] reported on Rivlin-Ericson Fluid through a Porous Medium in a Parallel Plate Channel. Veera Krishna and Dharmaiah [3] examined hall effects on MHD pulsatile flow through a porous medium in a flexible channel. Veerakrishna and Dharmaiah [4] analyzed heat transfer on unsteady MHD couette flow of a bingham fluid through a porous medium taking hall currents. Dharmaiah et al. [5] studied those effects of radiation, chemical reaction and soret on unsteady mhd free convective flow over a vertical porous plate. Ramprasad et al. [6] discussed unsteady mhd convective heat and mass transfer flow past an inclined moving surface with heat absorption. Charan Kumar et al. [7] studied chemical reaction and soret effects on casson mhd fluid over a vertical plate. Vedavati et al. [8] reviewed chemical reaction, radiation and dufour effects on casson magneto hydro dynamics fluid flow over a vertical plate with heat source /sink. Babyrani et al. [9] investigated synthetic response and radiation impacts on unsteady mhd free convective flow over a vertical permeable plate. Balamurugan et al. [10] surveyed mhd free convective flow past a semi-infinite vertical permeable moving plate with heat absorption. Dharmaiah et al. [11] considered effect of chemical reaction on mhd casson fluid flow past an inclined surface with radiation. Dharmaiah [12] experimented an unsteady magneto hydro dynamic heat transfer flow in a rotating parallel plate channel through a porous medium with radiation effect. Dharmaiah et al. [13] expertised magneto hydro dynamics convective flow past a vertical porous surface in slip-flow regime. Dharmaiah et al. [14] carried out effect of chemical reaction on mhd casson fluid flow past an inclined surface with radiation. Babyrani et al. [15] tested mhd transient free convection aligned magnetic and chemically reactive flow past a porous inclined plate with radiation and temperature gradient dependent heat source in slip flow regime. Dharmaiah et al. [16] presented the effect of chemical reaction on heat and mass transfer mhd flow Ag, TiO_2 and Cu water nano fluids over a semi infinite surface. Balamurugan et al. [17] overviewed influence of radiation absorption, viscous and joules dissipation on mhd free convection chemically reactive and radiative flow in a moving inclined porous plate with temperature dependent heat source. Baby Rani [18] examined influence of radiation on heat and mass transfer in mhd fluid flow over an infinite vertical porous surface with chemical reaction. In this paper, we discuss the steady hydro magnetic flow of an incompressible couple stress fluid in a parallel plate channel bounded on one side by a porous bed. Results are discussed through graphs.

II. FORMULATION AND SOLUTION OF THE PROBLEM

We consider the steady flow of a couple stress fluid in a parallel plate channel bounded on one side by a porous bed.

- ✓ The fluid is driven by a uniform pressure gradient parallel to the channel plates
- ✓ The entire flow field is subjected to a uniform inclined magnetic field of strength H_o inclined at an angle of inclination α
- ✓ The normal to the boundaries in the transverse *xy*-plane.

The equations of the motion for the incompressible polar fluids are

$$\nabla \cdot V = 0$$

$$\rho \frac{DV}{Dt} = -\nabla P + \mu \nabla^2 V - J \times B - \eta (\nabla^2)^2 V$$

We choose a Cartesian system O(*x, y, z*) such that the boundary walls are at $z=0$ and $z=l$.

The steady hydro-magnetic equations governing the couple stress fluid under the influence of a uniform inclined magnetic field with reference to a frame and The Brinkman equations governing flow through porous medium with respect to the frame are

$$\frac{\eta}{\rho}\frac{d^4u}{dz^4} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu \frac{d^2u}{dz^2} - \frac{\sigma \mu_e{}^2 H_0{}^2 Sin^2\alpha}{\rho} u \tag{1}$$

$$\frac{\eta}{\rho}\frac{d^4w}{dz^4} = \nu \frac{d^2w}{dz^2} - \frac{\sigma \mu_e{}^2 H_0{}^2 Sin^2\alpha}{\rho} w \tag{2}$$

$$\frac{\eta}{\rho}\frac{d^4u_p}{dz^4} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu_{eff} \frac{d^2u_p}{dz^2} - \frac{\sigma \mu_e{}^2 H_0{}^2 Sin^2\alpha}{\rho} u_p - \frac{\nu}{k} u_p \tag{3}$$

$$\frac{\eta}{\rho}\frac{d^4w_p}{dz^4} = \nu_{eff} \frac{d^2w_p}{dz^2} - \frac{\sigma \mu_e{}^2 H_0{}^2 Sin^2\alpha}{\rho} w_p - \frac{\nu}{k} w_p \tag{4}$$

Let $q = u + iw \,, q_p = u_p + iw_p$

Now combining equations (1) and (2), we obtain

$$\frac{\eta}{\rho}\frac{d^4q}{dz^4} - \nu \frac{d^2q}{dz^2} + \left(\frac{\sigma \mu_e{}^2 H_0{}^2 Sin^2\alpha}{\rho}\right) q = -\frac{1}{\rho}\frac{\partial p}{\partial x} \tag{5}$$

combining equations (3) and (4), we obtain

$$\frac{\eta}{\rho}\frac{d^4q_p}{dz^4} - \nu_{eff} \frac{d^2q_p}{dz^2} + \left(\frac{\sigma \mu_e{}^2 H_0{}^2 Sin^2\alpha}{\rho} + \frac{\nu}{k}\right) q_p = -\frac{1}{\rho}\frac{\partial p}{\partial x} \tag{6}$$

The boundary conditions are,

$$q_p = 0 \;, \qquad at \;\; z = 0 \tag{7}$$

$$q = 0 \;, \qquad at \;\; z = l \tag{8}$$

$$\frac{d^2q_p}{dz^2} = 0 \,, \qquad at \;\; z = 0 \tag{9}$$

$$\frac{d^2q}{dz^2} = 0 \;, \qquad at \;\; z = l \tag{10}$$

The interfacial conditions are

$$\left.\begin{aligned} & q = q_p \\ & \nu \frac{dq}{dz} = \nu_{eff} \ \frac{dq_p}{dz} \\ & \nu \frac{d^2 q}{dz^2} = \nu_{eff} \ \frac{d^2 q_p}{dz^2} \\ & \nu \frac{d^3 q}{dz^3} = \nu_{eff} \ \frac{d^3 q_p}{dz^3} \end{aligned}\right\} \quad at \qquad z = h \tag{11}$$

We introduce the non-dimensional variables

$$z^* = \frac{z}{l} \ ,q^* = \frac{ql}{\nu} \ , \quad q_p{}^* = \frac{q_p \, l}{\nu} ,P^* = \frac{Pl^2}{\rho \nu^{\,2}} , \quad h^* = \frac{h}{l}$$

The solution of the problem using the governing non-dimensional equations are

$$q = D_1 e^{m_1 z} + D_2 e^{m_2 z} + D_3 e^{-m_1 z} + D_4 e^{-m_2 z} + \frac{P}{M^2 Sin^2 \alpha} \tag{12}$$

$$q_p = D_5 e^{m_5 z} + D_6 e^{m_6 z} + D_7 e^{-m_5 z} + D_8 e^{-m_6 z} + \frac{P}{M^2 Sin^2 \alpha + D^{-1}} \tag{13}$$

III. RESULTS AND DISCUSSIONS

The profiles for the velocity components *u* and *v* are drawn for the variations in the governing parameters *M*, D^{-1} and *S* and for varying thickness of the porous bed with fixed angle of inclination. Figures (1-3) represent the velocity profiles for *u* related to small thickness of the porous bed while figures (4-6) correspond to the velocity profiles for *v*. The effect of the magnetic field may be observed in fig (1 and 4) for small thickness of the porous bed. We find that in the clean fluid region the resultant velocity experiences a retardation for higher intensity of the magnetic field, although in the porous region *u* reduces where as *v* enhances with *M*. In the clean fluid region as increase in *M* enhances *u* increases where as *v* reduces with *M*. The resultant velocity also reduces with M in the entire flow region. The variation with reference to D^{-1} is much similar with respect to *u* and *v* reducing in the clean fluid region for increase in D^{-1}. However *u* retards in the entire flow region and *v* enhances in the porous region the increase in D^{-1}(Fig 2 and 8). The resultant velocity retards with D^{-1} in the entire fluid region. We observe that an increase in *S* enhances both *u* and *v* in the porous region. The rate of enhancement with reference to *u* is larger than the corresponding the growth in *v* in the clean fluid region (Fig 3-6). The resultant velocity also enhances with *S* in the entire flow region. The behaviour of the velocity components in the clean fluid region as well as porous region with variations in *M*, D^{-1} and *S* remains un effected when the thickness of the porous bed slightly increases. The Figures (7-12) correspond to the velocity profiles for different variations in governing parameters in case of larger thickness of the porous bed (h=0.8). It is evident from fig (7 and 10) that the thickness of the porous bed does not affect the behaviour of the fluid flow with reference to the variation in the intensity of the magnetic field. Irrespective thickness of the porous bed, lower the permeability of the porous bed lesser the magnitude of the velocity components *u* and *v* as well as the resultant velocity (Fig. 8 and 11). The influence of the thickness of the bed on the flow field is quite pronouncing (Glary) is evident from the behaviour of the velocity components with variations in the couple stress parameter *S*. In contrast to the smaller thickness of the case, we find that the velocity components as well as the resultant velocity experiences in the entire flow field for increasing in the couple stress parameter *S* (Fig 9 and 12). *i.e.*, Likewise *u*, *v* and the resultant velocity increases in its magnitude with increase in *S* in the porous bed in contrast to the small thickness case.

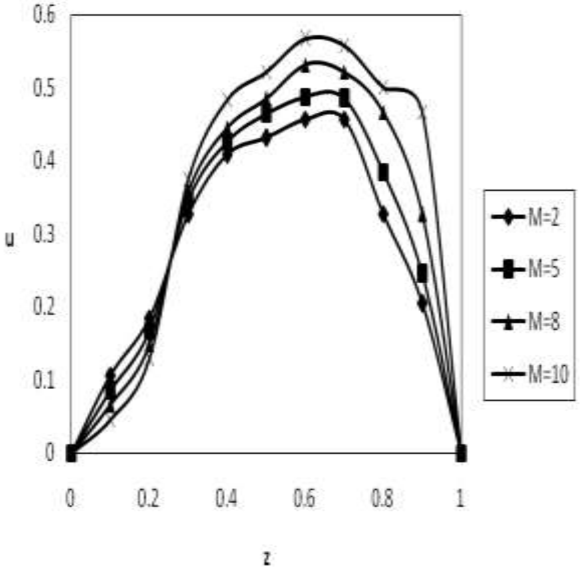

Figure 1. The velocity profile *u* for different *M* .

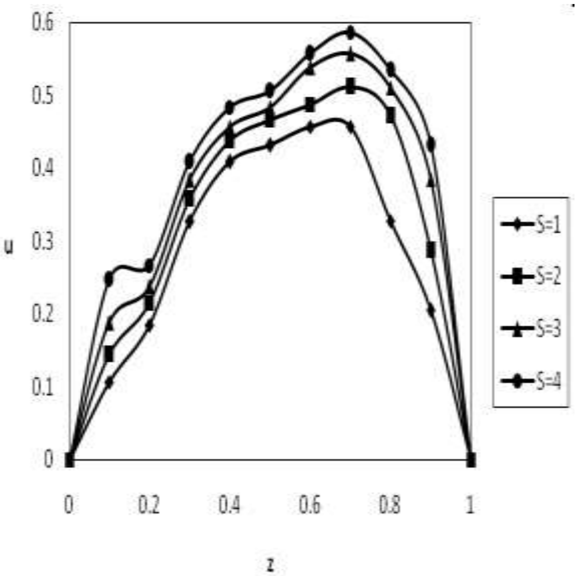

Figure 3. The velocity profile *u* for different *S* .

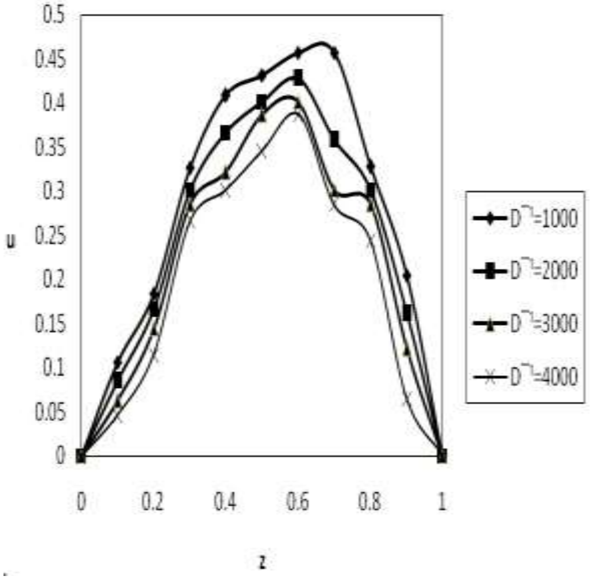

Figure 2. The velocity profile *u* for different D^{-1} .

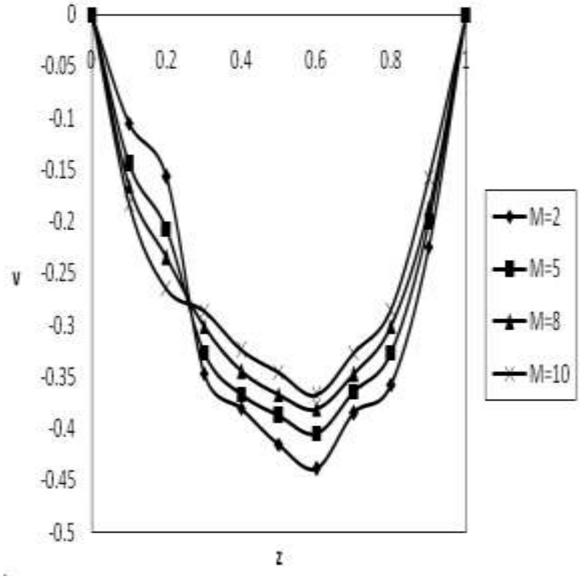

Figure 4. The velocity profile *v* for different *M* .

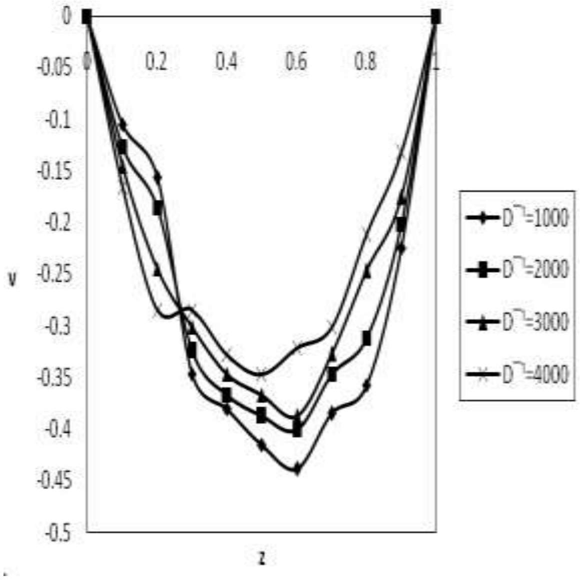

Figure 5. The velocity profile v for different D^{-1} .

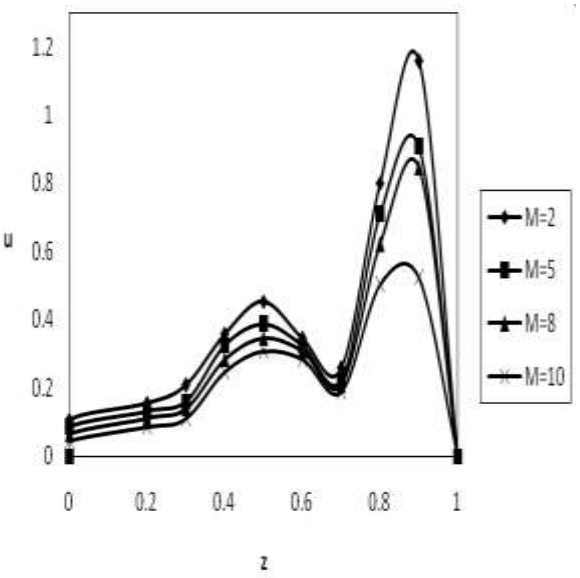

Figure 7. The velocity profile u for different M .

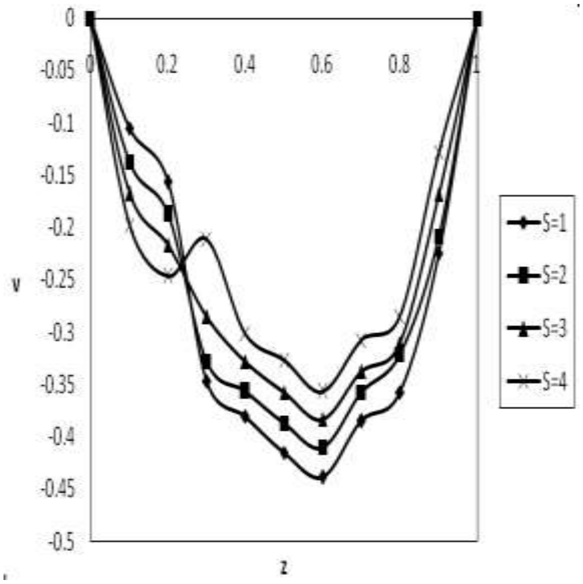

Figure 6. The velocity profile v for different S .

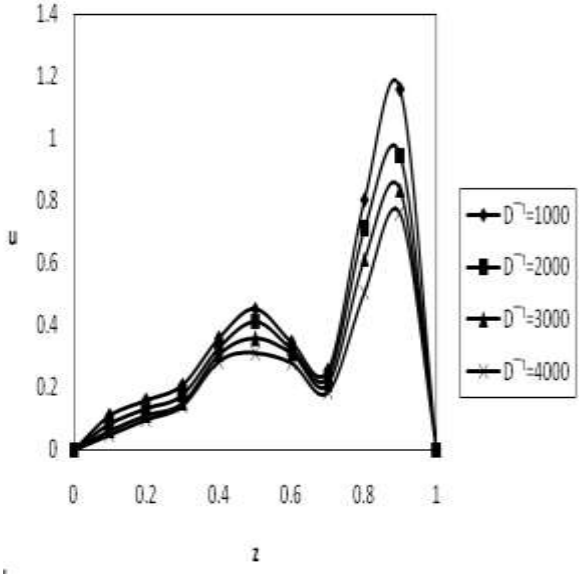

Figure 8. The velocity profile u for different D^{-1}.

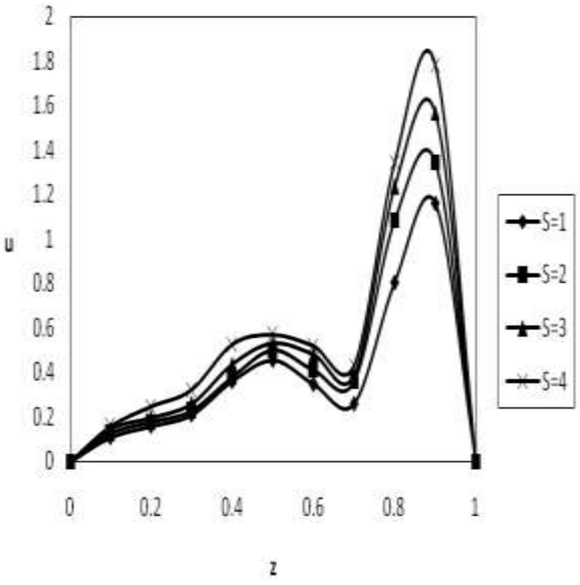

Figure 9. The velocity profile *u* for different *S* .

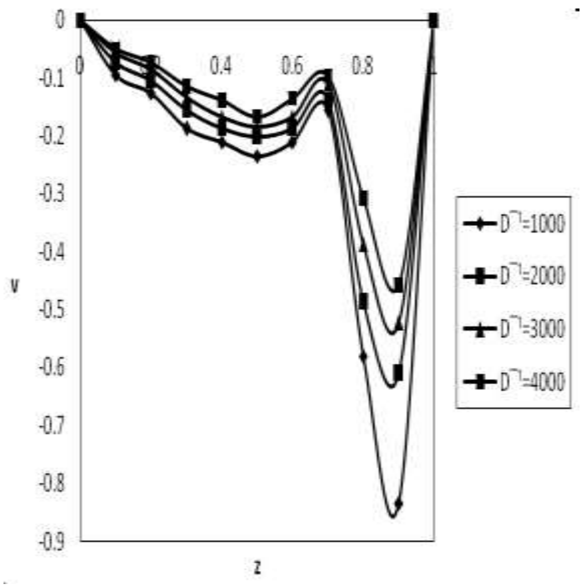

Figure 11. The velocity profile *v* for different D^{-1} .

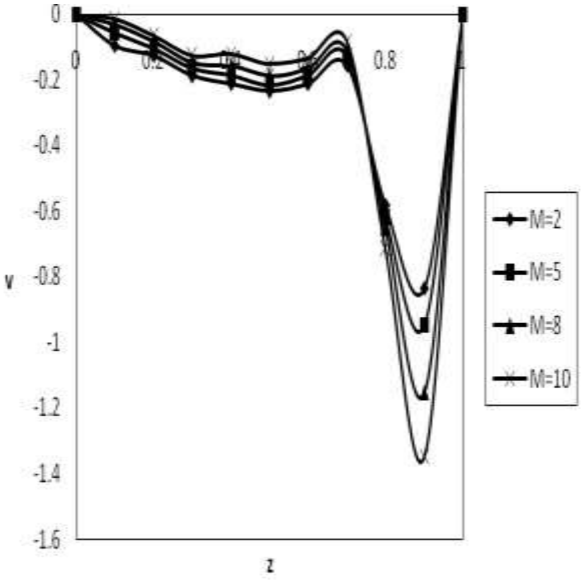

Figure 10. The velocity profile *v* for different *M* .

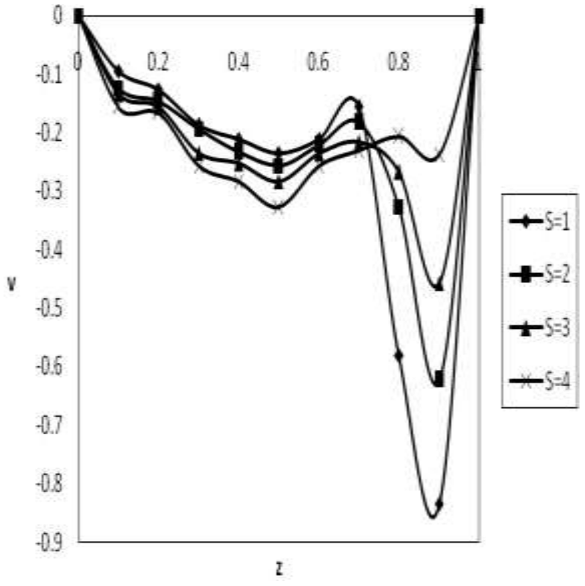

Figure 12. The velocity profile *v* for different *S*.

REFERENCES

[1] G.Dharmaiah and M. Veera Krishna, "Finite difference analysis on mhd free convection flow through a porous medium along a vertical wall", Asian Journal of Current Engineering and Maths, vol. 2, pp.273-280, 2013.

[2] M. Veera Krishna and G. Dharmaiah, "MHD flow of a rivlin-ericson fluid through a porous medium in a parallel plate channel under externally applied boundary acceleration", International Journal of Engineering Inventions, vol. 2, pp. 34-40, 2013.

[3] M. Veera Krishna and G. Dharmaiah, "Hall effects on MHD pulsatile flow through a porous medium in a flexible channel", International Journal of Advances in Engineering & Technology, vol. 6, pp. 1552-1563, 2013.

[4] M. veerakrishna and G. Dharmaiah, "Heat transfer on unsteady MHD couette flow of a bingham fluid through a porous medium in a parallel plate channel with uniform suction and injection under the effect of inclined magnetic field and taking hall currents", IJAMA, vol. 5, pp. 147-163, 2013.

[5] G. Dharmaiah, N. Vedavathi, and K.S.Balamurugan, “Effects of radiation, chemical reaction and soret on unsteady mhd free convective flow over a vertical porous plate”, IJSIMR, vol. 3, pp. 93-101, 2015.

[6] J.L.Ramprasad, K.S.Balamurugan, and G. Dharmaiah, “Unsteady mhd convective heat and mass transfer flow past an inclined moving surface with heat absorption”, JP Journal of Heat and Mass Transfer, vol. 8, pp. 33-51, 2016.

[7] G. Charan Kumar, G. Dharmaiah, K.S. Balamurugan, and N. Vedavathi, “Chemical reaction and soret effects on casson mhd fluid over a vertical plate”, Int. J. Chem.Sci., vol. 14, pp. 213-221, 2016.

[8] N. Vedavati, G. Dharmaiah, K.S. Balamurugan, and G. Charan Kumar, “Chemical reaction, radiation and dufour effects on casson magneto hydro dynamics fluid flow over a vertical plate with heat source /sink”, Global Journal Of Pure And Applied Mathematics, vol. 7, pp. 191-200, 2016.

[9] Ch.Baby Rani, G. Dharmaiah, K.S.Balamurugan, and Sk.Mohiddin Shaw, “Synthetic response and radiation impacts on unsteady mhd free convective flow over a vertical permeable plate”, Int. J. Chem.Sci., vol. 14, pp. 2051-2065, 2016.

[10] K.S.Balamurugan, G. Dharmaiah, S.V.K.Varma, and V.C.C.Raju, “MHD free convective flow past a semi-infinite vertical permeable moving plate with heat absorption”, International Journal of Engineering & Scientific Research, vol. 4, pp. 46-58, 2016.

[11] M.Venkateswarlu, D. Venkatalakshmi, and G.Dharmaiah, “Influence of slip condition on radiative mhd flow of a viscous fluid in parallel porous plate channel in presence of heat absorption and chemical reaction”, J.KSIAM, vol. 20, pp. 333-354, 2016.

[12] G.Dharmaiah, “An unsteady magneto hydro dynamic heat transfer flow in a rotating parallel plate channel through a porous medium with radiation effect”, Innovare Journal Of Engineering & Technology, vol. 5, pp. 34-37, 2017.

[13] G.Dharmaiah, Y.UdayKumar, and N.Vedavathi, “Magneto hydro dynamics convective flow past a vertical porous surface in slip-flow regime”, IJTAM, vol. 12, pp.71-81, 2017.

[14] G.Dharmaiah, K.S.Balamurugan, V.C.C.Raju, and N.Vedavathi, “Effect of chemical reaction on mhd casson fluid flow past an inclined surface with radiation”, SKIT Journal, vol. 7, pp:53-59, 2017.

[15] Ch.BabyRani, K.Ramaprasad, K.S.Balamurugan, and G. Dharmaiah, “MHD transient free convection aligned magnetic and chemically reactive flow past a porous inclined plate with radiation and temperature gradient dependent heat source in slip flow regime”, IOSR Journal of Mathematics, vol. 13, pp. 34-45, 2017.

[16] G.Dharmaiah, J.Prakash., K.S.Balamurugan, and N.Vedavathi, “The effect of chemical reaction on heat and mass transfer mhd flow Ag, Tio_2 and Cu water nano fluids over a semi infinite surface”, Global Journal Of Pure And Applied Mathematics, vol. 8, pp. 6609-6632, 2017.

[17] K.S. Balamurugan, J.L. Ramaprasad, G.Dharmaiah and V.C.C. Raju, “Influence of radiation absorption, viscous and joules dissipation on mhd free convection chemically reactive and radiative flow in a moving inclined porous plate with temperature dependent heat source”, International Refereed Journal of Engineering and Science, vol. 5, pp. 20-31, 2016.

[18] Ch.Baby Rani, Sk. Mohiddin Shaw, and G.Dharmaiah, “Influence of radiation on heat and mass transfer in mhd fluid flow over an infinite vertical porous surface with chemical reaction”, International Journal of Mathematics and its Applications, vol. 5, pp. 731-740, 2017.