Bianchi Type I Inflationary Cosmological Model with Bulk Viscosity in General Relativity

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Abstract - In present paper our intention to constructed locally symmetric Bianchi Type I space time undertaken in framework of massless scaler field with flat potential. In presence of bulk viscosity model leads to inflationary scenario of universe. Einstein field equation are solved by using suitable transformation yields negative deceleration parameter represent accelerating phase of universe. The model isotropize at special condition and spatial volume increase with time represent eternal inflation. In presence of bulk viscosity geometrical and physical aspect of model are discussed.

Keywords- Bianchi Type I space time, Bulk viscosity, Eternal inflation

1. INTRODUCTION

Many cosmological problems are being investigated by cosmologist to understand the early evolution of universe. Einstein theory of general relativity[1] has provided sophisticated theory of gravitation. Inflation means extremely rapid expansion of early universe by a factor of 10^{78} in volume driven by negative pressure vacuum energy density. Guth[2] first introduced the basic idea of inflation while face the problem we do not monopole exist today. He investigated the false vacuum with positive energy leads an expansion of space exponentially. Inflationary scenario of universe provides a potential solution to cosmological problem like Horizon problem, Isotropy, Homogeneity and Magnetic Monopole. Inflationary phenomenon for homogenous and isotropy space time (FRW model) is investigated by Linde[3], La and Steinhardth[4], Burd and Borrow[5]. Rothman and Ellis[6] have resulted out we have find solution of isotropic problem if we deals with anisotropic space time metric that isotropize in special case. Some Bianchi Type I inflationary model for flat potential in general relativity is constructed by Bali and Jain[7] and Bali[8]. Bali and Singh[9] provide LRS Bianchi Type I space time metric for stiff fluid with variable bulk viscosity. Bali and Poonia [10] studied Bianchi Type VI_0 cosmological model to study inflationary scenario of universe and observe late time acceleration. Naidu et.al.[11] constructed inflationary model in Kantowski –Sachs space with constant deceleration parameter. Many authors Zimdahl[12], Saha[13], Peebles[14], Sahni and Starobinski[15] discussed the cosmological phenomenon under consideration of bulk viscosity and study the effect on evolution. Khalatnikov and Belinskii[16] studied FRW model by taking bulk viscosity as function of energy density. Reddy[17] have investigated spatially, anisotropic and homogenous Bianchi Type V space time to study inflation and structure of early universe.

In this paper, we have presented a LRS Bianchi Type I inflationary cosmological model with bulk viscosity in presence of flat potential to discuss inflationary scenario of universe. Spatial volume increase with time which represent accelerating phase of universe. Model isotropize at special case. This study will provide some sufficient fact to study the astrophysical phenomenon. Negative deceleration parameter yield de-sitter universe. Geometrical and physical significance of model are discussed.

II. THE METRIC AND FIELD EQUATIONS

We consider LRS Bianchi Type I metric in the form

$$ds^2 = -dt^2 + a^2 dx^2 + b^2 (dy^2 + dz^2) \tag{1}$$

Where a and b are metric potential and function of parameter t only

Gravitational field minimally coupled to scalar field $V(\phi)$ is given by

$$l = \int \sqrt{-g}\left(R - \frac{1}{2} g^{ij} \phi,_i \phi,_j - V(\phi) \right) dx^4 \tag{2}$$

The Einstein field equation $(8\pi G = c = 1)$ for massless less scalar field ϕ is given by

$$R_{ij} - \frac{1}{2} R g_{ij} = -T_{ij} \tag{3}$$

Guth provide energy momentum tensor T_{ij} in presence of viscosity for scaler field is given by

$$T_{ij} = \partial_i \phi \partial_j \phi - \left[\frac{1}{2} \partial_h \phi \partial^h \phi + V(\phi) \right] g_{ij} - \xi\theta \left(g_{ij} + v_i v_j \right) \tag{4}$$

With

$$\frac{1}{\sqrt{-g}} \partial_i \left[\sqrt{-g} \partial^i \phi \right] = -\frac{dV}{d\phi} \tag{5}$$

Where V is potential, ϕ is Higgs field, ξ is coefficient of viscosity and θ is expansion scalar

The commoving derivative is considered as $v^i = (0,0,0,1)$ i.e. $v^1 = v^2 = v^3 = 0, \quad v^4 = 1$

Einstein field equation (3) for metric (1) become

$$\frac{b'^2}{b^2} + 2\frac{b''}{b} = \frac{1}{2}\phi'^2 - K - \alpha \tag{6}$$

$$\frac{a''}{a} + \frac{b''}{b} + \frac{a'b'}{ab} = \frac{1}{2}\phi'^2 - K - \alpha \tag{7}$$

$$\frac{b'^2}{b^2} + 2\frac{a'b'}{ab} = -\left[\frac{1}{2}\phi'^2 + K + 2\alpha \right] \tag{8}$$

For inflationary solution, we have assumed $V(\phi)$ = K, $\xi\theta$ = α

We consider $\xi\theta$ = α (constant) because it have important role to connect with occurrence of LR cosmology using FRW metric as given by Brevik et.al.[18]

From equation (5) $$\phi'' + \left(\frac{a'}{a} + 2\frac{b'}{b} \right)\phi' = 0 \tag{9}$$

On solving provide $$\phi' = \frac{\lambda}{ab^2}$$

(10)

From equation (6) and (7) we have

$$\frac{a'b'}{ab}+\frac{a''}{a}-\frac{b''}{b}-\frac{b'^2}{b^2}=0 \tag{11}$$

Which gives $$\left(\frac{a'}{a}-\frac{b'}{b}\right)'+\left(\frac{a'}{a}-\frac{b'}{b}\right)\left(\frac{a'}{a}+2\frac{b'}{b}\right)=0 \tag{12}$$

On integration we get $$\left(\frac{a'}{a}-\frac{b'}{b}\right)=\frac{\mu}{ab^2} \tag{13}$$

Now from equation (6), (7) and (8) we have

$$\left(\frac{a'}{a}+2\frac{b'}{b}\right)=\frac{\lambda^2}{a^2b^4}-K-\frac{1}{2}\alpha \tag{14}$$

Assume $ab^2=\eta$ in equation (13) and provide eq. (17)

$$\frac{a'}{a}-\frac{b'}{b}=\frac{\mu}{\eta} \tag{15}$$

Equation (14) become

$$\left(\frac{\eta'}{\eta}\right)'+\frac{a'^2}{a^2}+2\frac{b'^2}{b^2}=\frac{\lambda^2}{\eta^2}-K-\frac{1}{2}\alpha \tag{16}$$

$$\frac{a'}{a}+2\frac{b'}{b}=\frac{\eta'}{\eta} \tag{17}$$

Equation (15) and (17) gives relation

$$3\frac{a'}{a}=\frac{\eta'}{\eta}+2\frac{\mu}{\eta} \tag{18}$$

and $$3\frac{b'}{b}=\frac{\eta'}{\eta}-\frac{\mu}{\eta}$$

(19)

Using Equation (18),(19) in (16) we obtained

$$\left(\frac{\eta'}{\eta}\right)'+\frac{1}{9}\left(\frac{\eta'}{\eta}+2\frac{\mu}{\eta}\right)+\frac{2}{9}\left(\frac{\eta'}{\eta}-\frac{\mu}{\eta}\right)^2=\frac{\lambda^2}{\eta^2}-K-\frac{1}{2}\alpha \tag{20}$$

On solving gives

$$\eta\eta''-\frac{2}{3}\eta'^2+K\eta^2+\frac{1}{2}\alpha\eta^2+\frac{2}{3}\mu^2-\lambda^2=0 \tag{21}$$

Let use the transformation

$$\eta'=\psi(\eta)$$

So that $$\eta''=\psi\frac{d\psi}{d\eta}$$

Equation (21) become

$$2\psi\frac{d\psi}{d\eta}-\frac{4}{3}\frac{\psi^2}{\eta}=(2K+\alpha)\eta+\frac{2\lambda^2-\frac{4}{3}\mu^2}{\eta} \tag{22}$$

Provide $$\eta'^2=\frac{3}{2}\eta^2(2K+\alpha)-\frac{3}{4}(2\lambda^2-\frac{4}{3}\mu^2)+D\eta^{\frac{4}{3}} \tag{23}$$

D is the constant of integration.

Since Equation (6),(7),(8) and (9) have Four unknown a,b,ϕ *and* $K=V(\phi)$

For find solution take $D=0$, which gives

$$\frac{d\eta}{dt}=\sqrt{\frac{3}{2}\eta^2(2K+\alpha)-\frac{3}{4}(2\lambda^2-\frac{4}{3}\mu^2)} \tag{24}$$

Leads to $$\frac{d\eta}{\sqrt{\frac{3}{2}(2K+\alpha)}\sqrt{\eta^2+\beta^2}}=dt$$

(25)

Where $$\beta^2=\frac{\mu^2-\frac{3}{2}\lambda^2}{\frac{3}{2}(2K+\alpha)} \tag{26}$$

gives $\eta=\beta\sinh\sqrt{\frac{3}{2}(2K+\alpha)}(t+\delta)$ where δ is constant of integration (27)

and

$$\eta'=\beta\sqrt{\frac{3}{2}(2K+\alpha)}\cosh\sqrt{\frac{3}{2}(2K+\alpha)}(t+\delta) \tag{28}$$

From Equation (18), (27) and (28) we get

$$3\frac{a'}{a}=\frac{\eta'}{\eta}+2\frac{\mu}{\eta}=\frac{\beta\sqrt{\frac{3}{2}(2K+\alpha)}\cosh\sqrt{\frac{3}{2}(2K+\alpha)}(t+\delta)+2\mu}{\beta\sinh\sqrt{\frac{3}{2}(2K+\alpha)}(t+\delta)} \tag{29}$$

$$3\frac{b'}{b}=\frac{\eta'}{\eta}-\frac{\mu}{\eta}=\frac{\beta\sqrt{\frac{3}{2}(2K+\alpha)}\cosh\sqrt{\frac{3}{2}(2K+\alpha)}(t+\delta)-\mu}{\beta\sinh\sqrt{\frac{3}{2}(2K+\alpha)}(t+\delta)} \tag{30}$$

On integrating we get

$$a=\tau^{\frac{1}{3}}\sinh^{\frac{1}{3}}\left[\sqrt{\frac{3}{2}(2K+\alpha)}(t+\delta)\right]\tanh^{\frac{2\mu}{\beta\sqrt{\frac{3}{2}(2K+\alpha)}}}\left[\frac{1}{2}\sqrt{\frac{3}{2}(2K+\alpha)}(t+\delta)\right] \tag{31}$$

From Equation (19), (27) and (28) we get

$$b=\gamma^{\frac{1}{3}}\sinh^{\frac{1}{3}}\left[\sqrt{\frac{3}{2}(2K+\alpha)}(t+\delta)\right]\tanh^{\frac{-\mu}{3\beta\sqrt{\frac{3}{2}(2K+\alpha)}}}\left[\frac{1}{2}\sqrt{\frac{3}{2}(2K+\alpha)}(t+\delta)\right] \tag{32}$$

Where τ and γ are the constant of integration

From Equation (10)

$$\phi^{'}=\frac{\lambda}{ab^2}=\frac{\lambda}{\beta\sinh\sqrt{\frac{3}{2}(2K+\alpha)}(t+\delta)} \tag{33}$$

Using the transformation $t+\delta=T$, $\tau^{\frac{1}{3}}x=X$, $\gamma^{\frac{1}{3}}y=Y$ and $\gamma^{\frac{1}{3}}z=Z$ we get the metric in the form

$$ds^2=-dT^2+\sinh^{\frac{2}{3}}\left[\sqrt{\frac{3}{2}(2K+\alpha)}T\right]\tanh^{\frac{2\mu}{3\beta\sqrt{\frac{3}{2}(2K+\alpha)}}}\left[\frac{1}{2}\sqrt{\frac{3}{2}(2K+\alpha)}T\right]dX^2$$
$$+\sinh^{\frac{2}{3}}\left[\sqrt{\frac{3}{2}(2K+\alpha)}T\right]\tanh^{\frac{-\mu}{3\beta\sqrt{\frac{3}{2}(2K+\alpha)}}}\left[\frac{1}{2}\sqrt{\frac{3}{2}(2K+\alpha)}T\right](dY^2+dZ^2) \tag{34}$$

III. PHYSICAL AND GEOMETRICAL PROPERTIES

Scalar of expansion $\theta=\frac{a^{'}}{a}+2\frac{b^{'}}{b}=\frac{\eta^{'}}{\eta}=\sqrt{\frac{3}{2}(2K+\alpha)}\coth\left[\sqrt{\frac{3}{2}(2K+\alpha)}T\right]$ (35)

Shear is given as

$$\sigma=\frac{1}{\sqrt{3}}\left(\frac{a^{'}}{a}-\frac{b^{'}}{b}\right)=\frac{1}{\sqrt{3}}\frac{\mu}{\beta\sinh\sqrt{\frac{3}{2}(2K+\alpha)}T} \tag{36}$$

Hubble Parameters in X, Y and Z direction is given as

$$H_1=\frac{\beta\sqrt{\frac{3}{2}(2K+\alpha)}\cosh\sqrt{\frac{3}{2}(2K+\alpha)}T+2\mu}{3\beta\sinh\sqrt{\frac{3}{2}(2K+\alpha)}T} \tag{37}$$

and

$$H_2=H_3=\frac{\beta\sqrt{\frac{3}{2}(2K+\alpha)}\cosh\sqrt{\frac{3}{2}(2K+\alpha)}T-\mu}{\beta\sinh\sqrt{\frac{3}{2}(2K+\alpha)}T} \tag{38}$$

Also $\frac{\sigma}{\theta}=\frac{\mu}{\sqrt{3}\eta\sqrt{\frac{3}{2}(2K+\alpha)}\cosh\sqrt{\frac{3}{2}(2K+\alpha)}T}$ (39)

Higgs field is given by

$$\phi=\int\frac{\lambda}{\beta\sinh\sqrt{\frac{3}{2}(2K+\alpha)}T}+N \tag{40}$$

Spatial Volume V is obtained as

$$V = \beta \sinh \sqrt{\frac{3}{2}(2K+\alpha)T} \tag{41}$$

Deceleration Parameter is given as

$$q = -\left(1 + 3\sec h^2 \left(\sqrt{\frac{3}{2}(2k+\alpha)T}\right)\right) \tag{42}$$

IV. CONCLUSION

The Spatial volume increase with time. When T→ ∞ then Spatial Volume (R^3) tends to infinity, it represent inflationary scenario of universe in Bianchi Type I model contain massless scalar field with flat potential. The expansion (θ) tends to infinite when T tends to zero and θ→ finite term when T tends to infinity. The physical quantity $\frac{\mu}{\beta}$ measures the anisotropy in the model. Shear (σ) tends to zero when T tends to infinity. Rate of Higgs field decrease with time. The model (34) start with a big-bang at T=0 .The deceleration parameter (q) is negative represent accelerating phase of universe. Since $\frac{\sigma}{\theta} \rightarrow 0$ for large value of T it shows that the model approaches isotropy. The Hubble parameter decrease with time.

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