# New Sort of Generalized Closed Sets in Topological Spaces

[1] R.Rajasubramaniam, [2] B.Meera Devi

[1] Assistant Professor,Department of Mathematics,
Arulmigu Palaniandavar College of Arts and Culture,
Palani-624601, Tamil Nadu, India.

[2]Assistant Professor,Department of Mathematics,
Sri S. Ramasamy Naidu Memorial College,
Sattur-626203, Tamil Nadu, India.

**Abstract**

This paper is devoted to introduce and study a new class of generalized closed sets, namely δg¨- closed sets and δg¨- open sets in Topological spaces. We prove that this class of δg¨-closed sets lies between the class of δ -closed sets and the class of g - closed sets. Also we find some relations between δg¨- closed sets and already existing closed sets. Further we discuss the characterisation and obtained their applications.
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## 1Introduction:

The concept of generalized closed sets plays a significant role in topology. In 1970, Levine [4] introduce the concept of generalized closed sets in topological space and a class of topological spaces called $T_{1/2}$ space. Extensive research on generalizing closedness was done in recent years by many Mathematicians. Arya and Nour [1], Maki et al [6], Veerakumar [11] and Lellis Thivagar et al [5] introduced generalized semi-closed sets, α-generalized closed sets, gˆ-closed sets and δgˆ-closed sets in topological spaces.

The purpose of this present paper is to define a new class os closed sets called δg¨-closed sets and also we obtain the basic properties of δg¨-closed seta in topological spaces. Applying this set, we obtain the new type of space which is called $T_{\delta \ddot{g}}$ space.

## 2.Preliminaries

Throughout this paper (X,τ ) (or simply X) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of X, cl(A), int(A) and $A^c$ denote the closure of A, the interior of A and the complement of A respectively. Let us recall the following definitions, which are useful in the sequel.

**Definition 2.1.** A subset A of a space (X,τ ) is called a

*(i)* semi-open set [3] if A ⊆ cl(int(A)).

*(ii)* pre-open set [8] if A⊆ int(cl(A)).

*(iii)* α-open set [9] if A⊆ int(cl(int(A))).

The complement of a semi-open (resp.pre-open,α-open) set is called semi-closed (resp. pre-closed,α-closed).

**Definition 2.2.** The δ -interior [12] of a subset A of X is the union of all regular open set of X contained in A and is denoted by $Int_\delta(A)$. The subset A is called δ -open [12] if A $=Int_\delta(A)$, i.e. a set is δ -open if it is the union of regular open sets. the complement of a δ -open is called δ -closed. Alternatively, a set A⊆ (X,τ ) is called δ -closed [12] if $A=cl_\delta(A)$, where $cl_\delta(A)$={ x∈X: int(cl(U))∩A=6 φ,U∈ τ and x∈ U}.

**Definition 2.3.** A subset A of (X,τ ) is called

*(i)* generalized closed (briefly g -closed) set [4] if cl(A)⊆U whenever A ⊆U and U is open set in (X,τ ).

*(ii)* generalized semi-closed (briefly gs-closed) set [1] if scl(A)⊆U whenever A ⊆U and U is open set in (X,τ ).

*(iii)* semi-generalized closed (briefly sg -closed) set [2] if scl(A)⊆U whenever A ⊆U and U is semi-open set in (X,τ ).

*(iv)* α- generalized closed (briefly αg -closed) set [6] if αcl(A)⊆U whenever A ⊆U and U is open set in (X,τ ).

*(v)* generalized α- closed (briefly gα-closed) set [6] if αcl(A)⊆U whenever A ⊆U and U is α-open set in (X,τ ).

*(vi)* gp- closed set [7] if pcl(A) ⊆ U whenever A ⊆ U and U is open set in (X,τ ).

*(vii)* gˆ-closed set [11] if cl(A)⊆U whenever A ⊆U and U is a semi-open set in (X,τ).

*(viii)* δ -gˆ-closed (briefly δgˆ-closed) set [5] if $cl_\delta(A)$ ⊆U whenever A ⊆U and U is a gˆ- open set in (X,τ ).

*(ix)* g¨-closed set [10] if cl(A) ⊆ U whenever A ⊆ U and U is sg -open in

(X,τ).

The complement of a g -closed (resp. gs-closed, sg -closed, αg -closed, gαclosed, gp-closed, g^-closed, δg^-closed and g¨-closed) set is called g -open (resp. gs-open, sg -open, αg -open,gα-open,gp-open, g^-open, δg^-open and g¨-open).

**Definition 2.4.** A space (X,τ ) is called

*(i)* $T_{1/2}$ -space [4] if every g-closed set in it is closed.

*(ii)* $T\hat{}_{3/4}$ -space [5] if every δg^-closed set in it is δ -closed.

## 3.Comparison

In this section we introduce δg¨- closed sets in topological spaces and study some relation between δg¨ -closed sets and other existing closed sets.

**Definition 3.1.** A subset A of (X,τ) is called δg¨-closed set if $cl_\delta(A) \subseteq U$ whenever $A \subseteq U$ and U is g¨-set.

The complement of δg¨-closed set is called δg¨-open set.

**Theorem 3.2.** Every δ -closed set in X is δg¨-closed set.

Proof. Let A be δ-closed set in X . Let U be any g¨- open set such that A ⊆ U . Since A is δ-closed, $cl_\delta(A) = A$ for every subset A of (X,τ). Thus $cl_\delta(A) \subseteq U$ and hence A is δg¨-closed.

**Remark 3.3.** The converse of Theorem 3.2, is not necessarily true as seen from the following example.

**Example 3.4.** Let X = {a,b,c} and τ = {φ,{a},{b,c},X}. Then the set {a,b} is δg¨-closed in (X,τ) but not δ -closed.

**Theorem 3.5.** Every δg^-closed set is δg¨-closed.

Proof. Let A be a δg^-closed set and U be any g¨- open set containing A. Since every g¨-open set is g^-open and A is δg^-closed, $cl_\delta(A) \subseteq U$ , whenever A ⊆ U , U is g¨-open. Hence A is δg¨-closed set.

**Remark 3.6.** The converse of the above theorem is not true in general as seen from the following example.

**Example 3.7.** Let X = {a,b,c} with the topology τ = {φ,{c},{a,b},X}. Then {a,c} is δg¨-closed but not δg^-closed set.

**Theorem 3.8.** Every δg¨-closed set is g -closed.

Proof. Let A be δg¨-closed set of (X,τ). Let U be open set in X such that A ⊆ U . Since every open set is g¨-open, so U is a g¨-open set of X . Since A is δg¨-closed set, $cl_\delta(A) \subseteq U$ . Also since $cl(A) \subseteq cl_\delta(A) \subseteq U$, we obtain that $cl(A) \subseteq U$ and hence A is g-closed.

**Remark 3.9.** The converse of the Theorem 3.8, is not true in general as seen form the following example.

**Example 3.10.** Let X = {a,b,c} and τ = {φ,{a},{a,b},{a,c},X}. Then the set A = {b} is g -closed but not δg¨-closed in (X,τ).

**Theorem 3.11.** Every δg¨-closed set in (X,τ) is αg -closed.

Proof. It is true that $cl_\alpha(A) \subseteq cl_\delta(A)$ for every subset A of X.

**Remark 3.12.** The converse of Theorem 3.11 is not necessarily true as seen from the following example.

**Example 3.13.** Let X = {a,b,c} and τ = {φ,{c},{a,c},{b,c},X}. Then the set A = {a} is αg -closed but not δg¨-closed.

**Theorem 3.14.** Every δg¨-closed set is gs-closed in (X,τ).

Proof. Let A be an δg¨-closed set and U be any open set containing A in (X,τ). Since every open set is g¨-open set and A is δg¨-closed, $cl_\delta(A) \subseteq U$ for every subset A of X. Also since $cl_s(A) \subseteq cl_\delta(A) \subseteq U$, we have $cl_s(A) \subseteq U$ and hence A is gs-closed.

**Remark 3.15.** A gs closed set need not be δg¨-closed as shown in the following example.

**Example 3.16.** Let X = {a,b,c} and τ = {φ,{b},{a,b},{b,c},X}. Then the set A = {c} is gs-closed but not δg¨-closed in (X,τ).

**Theorem 3.17.** Every δg¨-closed set is gp-closed in (X,τ).

Proof. Let A be an δg¨-closed set and U be any open set containing A in (X,τ). Since every open set is g¨-open set and A is δg¨-closed, $cl_\delta(A) \subseteq U$ for every subset A of X. Also since $cl_p(A) \subseteq cl_\delta(A) \subseteq U$, we obtain $cl_p(A) \subseteq U$ and hence A is gp-closed.

**Remark 3.18.** The converse of the above Theorem in not true in general as seen from the following example.

**Example 3.19.** Let X = {a,b,c} with the topology τ = {φ,{a,b},X}. Then the set A = {a} is gp-closed but not δg¨-closed in (X,τ).

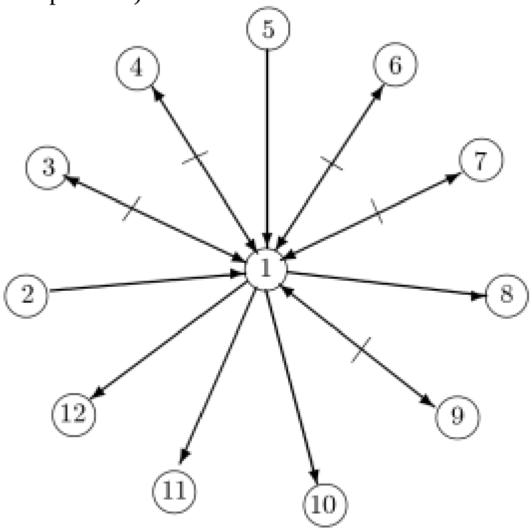
**Remark 3.20.** The following examples show that the δg¨-closeness is independent from semi-closeness, sg -closeness, gα-closeness, g^-closeness and g¨closeness.

**Example 3.21.** Let X = {a,b,c} with the topology τ = {φ,{a},{a,c},X}. Then the set A = {c} is semi-closed but δg¨-closed. The set B = {a,b} is δg¨-closed but not semi-closed.

**Example 3.22.** Let X = {a,b,c} with the topology τ = {φ,{b},{a,b},X}. Then the set A = {a} is both sg -closed and gα-closed but not δg¨-closed. The set B = {b,c} is δg¨-closed but neither sg -closed nor gα-closed.

**Example 3.23.** Let X = {a,b,c} with the topology τ = {φ,{c},{a,c},X}.
Then the set {b,c} is δg¨-closed but neither g^-closed nor g¨-closed. Also Let X = {a,b,c} with the topology τ = {φ,{a},{a,b},{a,c},X}. Then the A = {b} is both g^-closed and g¨-closed but not δg¨-closed.

**Remark 3.24.** We summarise the fundamental relationships between δg¨-closed and several types of generalized closed sets in the following diagram. Here A → B (resp. A = B ) represents A implies B but converse is not necessary true
(resp. A and B are independent)




1.δg¨-closed 2.δ-closed 3.semi-closed 4.sg-closed 5.δg^-closed 6.^g-closed
7.¨g-closed 8.g-closed 9.gα-closed 10.gp-closed 11.gs-closed 12.αg-closed

## 4.Properties of δg¨-closed sets

**Theorem 4.1.** The finite union of δg¨-closed sets in a space X is δg¨-closed.

Proof. Let $\{A_i/i = 1,2,...n\}$ be a finite class of δg¨-closed subsets of a space (X,τ). Then for each g¨-open set $U_i$ in X containing $A_i$, $cl_\delta(A_i) \subseteq U_i$, $i \in \{1,2,...n\}$. Hence $\cup_i A_i \subseteq \cup_i U_i = V$. SinceS V is g¨-open sets in (X,τ). Also $\cup_i cl_\delta(Ai) = cl_\delta(\cup_i A_i) \subseteq V$. Therefore $\cup_i A_i$ is δg¨-closed in (X,τ).

**Remark 4.2.** The intersection of two δg¨-closed sets need not be δg¨-closed sets.

**Example 4.3.** Let X = {a,b,c} with the topology τ = {φ,{b},X}. Then {a,b} and {b,c} are δg¨-closed sets, but {a,b}∩{b,c} = {b} is not δg¨-closed.

**Proposition 4.4.** If A is both g¨-open and δg¨-closed set of (X,τ), then A is δ -closed.

Proof. Let A be both g¨-open and δg¨-closed set of (X,τ). Then $cl_\delta(A) \subseteq A$ whenever A is g¨-open and $A \subseteq cl_\delta(A)$.Therefore we obtain that $A = cl_\delta(A)$ and hence A is δ-closed.

**Proposition 4.5.** If A is δg¨-closed set of (X,τ) such that $A \subset B \subset cl_\delta(A)$, then B is also a δg¨-closed set of (X,τ).

Proof. Let U be a g¨-open set of (X,τ) such that $B \subset U$. Since $A \subset B$, $A \subset U$. Also since A is δg¨-closed, we have $cl_\delta(A) \subset U$. Now $cl_\delta(B) \subset cl_\delta(cl_\delta(A)) = cl_\delta(A) \subset U$. Therefore B is a δg¨-closed set of (X,τ).

**Proposition 4.6.** Let A be δg¨-closed set of (X,τ), then $cl_\delta(A)-A$ does not contain a non-empty g¨-closed set.
Proof. Suppose that A is δg¨-closed, let F be a g¨-closed set contained in $cl_\delta(A) - A$. Now $F^c$ is a g¨-open set in X such that $A \subseteq F^c$. Since A is δg¨-closed set of (X,τ), then $cl_\delta(A) \subset F^c$. Hence $F \subset (cl_\delta(A))^c$. Also $F \subset cl_\delta(A) - A$. Therefore $F \subseteq (cl_\delta(A))^c \cap cl_\delta(A) = \varphi$. Hence F = φ.

**Theorem 4.7.** The intersection of a δg¨-closed set and δ -closed set is always δg¨-closed.

Proof. Let A be δg¨-closed and F be δ-closed. Let U be g¨-open set such that $A \cap F \subseteq U$, which implies $A \subseteq U \cup F^c$. Here $F^c$ is δ-open, so $F^c$ is open. Thus $F^c$ is g¨-open. Hence $U \cup F^c$ is g¨-open, so $cl_\delta(A) \subseteq U \cup F^c$. Now $cl_\delta(A \cap F) \subseteq cl_\delta(A) \cap F \subseteq U$. Hence $A \cap F$ is δg¨-closed.

**Theorem 4.8.** Let A be a δg¨-closed set of X. Then A is δ -closed iff $cl_\delta(A)-A$ is g¨-closed.

Proof. Necessity: Let A be δ-closed subset of (X,τ). Then $cl_\delta(A) = A$, so $cl_\delta(A) - A = \varphi$, which is g¨-closed.
Sufficiency: Since A is δg¨-closed, by Proposition 4.6, $cl_\delta(A) - A$ does not contain a non-empty g¨-closed set, which implies $cl_\delta(A) - A = \varphi$. That is, $cl_\delta(A) = A$. Hence A is δ-closed.

**Proposition 4.9.** In a topological space X , for each $x \in X$ either {x} is g¨-closed or $\{x\}^c$ is δg¨-closed in (X,τ).

Proof. Suppose that {x} is not g¨-closed in (X,τ). Then $\{x\}^c$ is not g¨-open and the only g¨-open set containing $\{x\}^c$ is the space X itself. Therefore $cl_\delta(\{x\}^c) \subseteq X$ so $\{x\}^c$ is δg¨-closed.

**Definition 4.10.** The intersection of all g¨-open subsets of X containing a set A is called the g¨-kernal of A and is denoted by g¨-ker(A).

**Lemma 4.11.** A subset A of (X,τ) is δg¨-closed iff $cl_\delta(A) \subseteq$ g¨-ker(A).

Proof. Assume that A is a δg¨-closed set in X. Then $cl_\delta(A) \subseteq U$ whenever $A \subseteq U$ and U is g¨-open in X. Let $x \in cl_\delta(A)$. Suppose $x \notin$ g¨-ker(A), then there is g¨-open U such that $x \notin U$. Since U is a g¨-open containing A, $x \notin cl_\delta(A)$ which is a contradiction. Hence $x \in$ g¨-ker(A). Conversely assume that $cl_\delta(A) \subseteq$ g¨-ker(A). If U is any g¨-open set containing A, then $cl_\delta(A) \subseteq$ g¨-ker(A)$\subseteq$ U. Therefore A is δg¨-closed.

**Definition 4.12.** The intersection of all δg¨-closed sets, each containing a set A in a topological space X is called the δg¨-closure of A and is denoted by δg¨-cl(A)

**Lemma 4.13.** Let A and B be subsets of (X,τ). Then

*(i)* δg¨-cl(φ) = φ and δg¨-cl(X) = X

*(ii)* If $A \subset B$ , then δg¨-cl(A) ⊂ δg¨-cl(B)

*(iii)* δg¨-cl(A) = δg¨-cl( δg¨-cl(A))

*(iv)* δg¨-cl(A ∪ B) = δg¨-cl(A)∪ δg¨-cl(B)

*(v)* δg¨-cl(A ∩ B) ⊂ δg¨-cl(A)∩ δg¨-cl(B)

**Remark 4.14.** If A is δg¨-closed in (X,τ), then δg¨-cl(A) = A but the converse is not true in general as shown in the following example.

**Example 4.15.** Let X = {a,b,c} with the topology τ = {φ,{a},X}. Let A = {a} then δg¨-cl(A) = {a} but {a} is not δg¨-closed set.

**Remark 4.16.** In general, δg¨-cl(A) ∩ δg¨-cl(B) * δg¨-cl(A∩B), as shown in the following example.

**Example 4.17.** Let X = {a,b,c} and τ = {φ,{a},{b},{a,b},{a,c},X}. Let A = {a,b}, B = {b,c}, then δg¨-cl(A)∩ δg¨-cl(B) = {b,c *{b} = δg¨-cl(A ∩ B).

## 5.δg¨-open sets in topological spaces

In this section we introduce the concept of δg¨-open sets in top[ological spaces and study some of their properties.

**Definition 5.1.** A subset A of a topological space (X,τ) is called δg¨-open if its complement $A^c$ is δg¨-closed in (X,τ).

**Theorem 5.2.** If a subset A of a topological space (X,τ) is δ -open then it is δg¨-open in X.

Proof. Let A be an δg¨-open set in X. Then $A^c$ is δg¨-closed in X. By Theorem 3.2, $A^c$ is δg¨-closed in (X,τ). Hence A is δg¨-open in X.

**Remark 5.3.** The converse of Theorem 5.2, is not true in general as seen in the following example.

**Example 5.4.** Let X = {a,b,c} and τ = {φ,{c},{b,c},X}, then the set {b,c} is δg¨-open but not δ -open in (X,τ).

**Proposition 5.5.** Every δg¨-open set is g -open, αg -open, gs-open and gpopen.

**Theorem 5.6.** A subset A of a topological space (X,τ) is δg¨-open iff $G \subseteq int_\delta(A)$ whenever G is g¨-closed set and $G \subseteq A$.

Proof. Necessity: Let A be $\delta\ddot{g}$-open. Then$A^c$ is $\delta\ddot{g}$- closed. Let G be a $\ddot{g}$closed set in $(X,\tau)$ contained in A. Then $G^c$ is a $\ddot{g}$-open in $(X,\tau)$ containing $A^c$. Since $A^c$ is $\delta\ddot{g}$-closed, $cl_\delta(A^c) \subseteq G^c$. Hence $G \subseteq int_\delta(A)$. Sufficiency: Assume that G ia contained in $int_\delta(A)$, whenever G is contained in A and G is $\delta\ddot{g}$-closed in $(X,\tau)$. Let $A^c \subseteq F$ where F is $\ddot{g}$-open. Then $F^c \subset A$. By hypothesis $F^c \subseteq int_\delta(A)$. This implies $cl_\delta(A^c) \subset F$. Thus is $\delta\ddot{g}$-closed. Hence A is $\delta\ddot{g}$-open.

**Remark 5.7.** For a subset A of X, $cl_\delta(X - A) = X - int_\delta(A)$.

**Proposition 5.8.** If A is a $\delta\ddot{g}$-open set in $(X,\tau)$ such that $int_\delta(A) \subseteq B \subseteq A$, then B is also a $\delta\ddot{g}$-open set of $(X,\tau)$.
Proof. Let $int_\delta(A) \subseteq B \subseteq A$ which implies that $X-A \subseteq X-B \subseteq X-int_\delta(A)$. By Remark 5.7, $X - A \subseteq X - B \subseteq cl_\delta(A)$. Since $X - A$ is $\delta\ddot{g}$-closed, by Proposition 4.5, $X - B$ is $\delta\ddot{g}$-closed and hence B is $\delta\ddot{g}$-open in $(X,\tau)$.

**Theorem 5.9.** If A and B are $\delta\ddot{g}$-open in X iff G = X whenever G is $\ddot{g}$-open and $X - A \subseteq X - B \subseteq int_\delta(A) \cup A^c \subseteq G$.

Proof. Necessity: Let A be $\delta\ddot{g}$-open set and G be $\ddot{g}$-open and $int_\delta(A) \cup A^c \subseteq$
G. This implies $G^c \subseteq (int_\delta(A) \cup A^c)^c = (int_\delta(A))^c \cap A = (int_\delta(A))^c - A^c =$
$cl_\delta(A^c) - A^c$. Since $A^c$ is $\delta\ddot{g}$-closed and $G^c$ is $\ddot{g}$-closed, it follows that $G^c = \phi$.
Hence $G = X$.

Sufficiency: Suppose that F is$\delta\ddot{g}$-closed and $F \subseteq A$. Then $int_\delta(A) \cup A^c \subseteq int_\delta(A) \cup F^c$. It follows by hypothesis that $int_\delta(A) \cup F^c = \varphi$ and hence $F \subseteq int_\delta(A)$. Hence by Theorem 5.6, A is $\delta\ddot{g}$-open in $(X,\tau)$.

**Lemma 5.10.** Let A be a subset of $(X,\tau)$ and $x \in X$. Then $x \in \delta\ddot{g}$-cl(A) iff $V \cap A$ =6 $\varphi$ for every $\delta\ddot{g}$-open set V containing x.

Proof. Suppose that there exists a $\delta\ddot{g}$-open set V containing x such that $V \cap A = \varphi$. Since $A \subset X - V$, $\delta\ddot{g}$-cl(A) $\subset X - V$ and hence $x \notin \delta\ddot{g}$-cl(A).
Conversely, assume that $x \notin \delta\ddot{g}$-cl(A). Then there exists a $\delta\ddot{g}$-closed set F containing A such that $x \notin F$. Since $x \in X - F$ and $X - F$ is $\delta\ddot{g}$-open, $(X - F) \cap A = \varphi$.

We introduce the following definition.

**Definition 5.11.** The union of all $\delta\ddot{g}$-open sets, each contained in a set A in a topological space X is called $\delta\ddot{g}$-interior of A and is denoted by $\delta\ddot{g}$-int(A).

**Lemma 5.12.** Let A and B be subsets of $(X,\tau)$. Then

*(i)* $\delta\ddot{g}$-int($\varphi$) = $\varphi$ and $\delta\ddot{g}$-int(X) = X

*(ii)* If $A \subseteq B$ , then $\delta\ddot{g}$-int(A) $\subseteq$ $\delta\ddot{g}$-int(B)

*(iii)* $\delta\ddot{g}$-int($A \cup B$) $\supseteq$ $\delta\ddot{g}$-int(A)$\cup$ $\delta\ddot{g}$-int(B)

*(iv)* $\delta\ddot{g}$-int($A \cap B$) $\subseteq$ $\delta\ddot{g}$-int(A)$\cap$ $\delta\ddot{g}$-int(B)

**Theorem 5.13.** Let A be any subset of $(X,\tau)$. Then the following statements hold.

*(i)* $(\delta\ddot{g}\text{-int}(A))^c = \delta\ddot{g}\text{-cl}(A^c)$

*(ii)* $\delta\ddot{g}\text{-int}(A) = (\delta\ddot{g}\text{-cl}(A^c))^c$

*(iii)* $\delta\ddot{g}\text{-cl}(A) = (\delta\ddot{g}\text{-int}(A^c))^c$

Proof. (i) Let $x \in (\delta\ddot{g}\text{-int}(A))^c$. Then $x \notin$ $\delta\ddot{g}$-int(A). That is every $\delta\ddot{g}$-open set U containing x is such that U * A. Hence $U \cap A^c \neq \varphi$. By lemma 5.10, $x \in \delta\ddot{g}\text{-cl}(A^c)$ and hence $(\delta\ddot{g}\text{-int}(A))^c \subseteq \delta\ddot{g}\text{-cl}(A^c)$. Now let $x \in \delta\ddot{g}\text{-cl}(A^c)$. Then by lemma 5.10, every $\delta\ddot{g}$-open set U containing x is such that $U \cap A^c \neq \varphi$. Thus $U \nsubseteq A$. This implies that $x \notin \delta\ddot{g}$-int(A). Therefore $x \in (\delta\ddot{g}\text{-cl}(A))^c$ and so $\delta\ddot{g}\text{-cl}(A^c) \subseteq (\delta\ddot{g}\text{-int}(A))^c$. Thus $(\delta\ddot{g}\text{-int}(A))^c = \delta\ddot{g}\text{-cl}(A^c)$.
(ii) Follows by taking complements in (i).
(iii) Follows by replacing A by $A^c$ in (i).

**6.Applications**

In this section we introduce a new type of space namely $T_{\delta\ddot{g}}$-space

**Definition 6.1.** A space X is called a $T_{\delta\ddot{g}}$ -space if every $\delta\ddot{g}$-closed set in it is $\delta$ -closed.

**Theorem 6.2.** For a topological space $(X,\tau)$, the following conditions are equivalent.

*(i)* $(X,\tau)$ is a $T_{\delta\ddot{g}}$ -space.

*(ii)* Every singleton of X is either $\delta\ddot{g}$-closed or $\delta$ -open.

Proof. (i) ⇒ (ii) Let x ∈X. Suppose that {x} is not a g¨-closed set of (X,τ). Then X −{x} is not g¨-open set. So X is the only g¨-open set of (X,τ) containing X −{x}. Thus X −{x} is an δg¨-closed set of (X,τ). Since (X,τ) is $T_{\delta g¨}$-space, X−{x} is an δ-closed set of (X,τ), and hence {x} is an δ-open set in (X,τ).

(ii) ⇒ (i) Let A be δg¨-closed set of (X,τ). Let x ∈ $cl_\delta$(A). By (ii), {x} is either δ-open or g¨-closed.

Case(a) : Let {x} be g¨-closed. If we assume that x ∉ A, then we would have $x \in cl_\delta(A) - A$ which can not be happen according to proposition 4.6. Hence $x \in A$.

Case(b) : Let {x} be δ-open. Since x ∈ $cl_\delta$(A) − A, then {x}∩ A ≠ φ. This shows that x ∈ A. So in both cases we have $cl_\delta$(A) ⊆ A. Trivially A ⊆ $cl_\delta$(A). Therefore A = $cl_\delta$(A) or equivalently A is δ-closed. Hence (X,τ) is $T_{\delta g¨}$-space.

**Theorem 6.3.** Every is $T_{\delta g¨}$ -space is $T\hat{}_{3/4}$ -space.

Proof. Let X be $T_{\delta g¨}$-space. Let A be δgˆ-closed set in X. Since every δgˆclosed set is δg¨-closed, A is δg¨-closed. By hypothesis A is δ-closed. Hence X is $T\hat{}_{3/4}$ -space.

**Remark 6.4.** The converse of the above theorem is not true in general as shown in the following example.

**Example 6.5.** Let X = {a,b,c} and τ = {φ,{a},{b,c},X}. Then the space (X,τ) is $T\hat{}_{3/4}$ -space but not $T_{\delta g¨}$ -space.

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