Commutativity of prime rings with symmetric biderivations on ideals

Dr. B. Ramoorthy Reddy
Assistant Professor, S V Engineering College, Karkambady road, Tirupati, Andhra Pradesh, India.
Kotte Amaranadha Reddy
Research Scholar, Vellore Institute of Technology, Vellore.
Dr. C. Jaya Subba Reddy
Assistant Professor, Dept.of Mathematics, S.V.University, Tirupati-517508, AndhraPradesh, India.

Abstract: A biadditive mapping $B_1(.,.): R \times R \to R$ is called a symmetric biderivationif $B_1(xy,z) = B_1(x,z)y + xB_1(y,z)$. Obviously, in this case also if $B_1(x,yz) = B_1(x,y)z + yB_1(x,z)$, for all $x,y,z \in R$. In the present paper, we initiated the commutativity of a prime ring R with a nonzero left ideal I of R, which satisfies certain conditions, namely $B_1(u,m)ou \in Z(R)$, $[B_1(u,m), B_1(v,m)] - B_1([u,v],m) \in Z(R)$, $B_1(u,m)oB_1(v,m) - B_1(uov.m) \in Z(R)$, $B_1([u,v],m) + [B_1(u,m),y] - [B_1(u,m),B_1(v,m)] \in Z(R)$, $B_1(uov,m) \pm [u,v] \in Z(R)$, $[B_1(u,m),B_1(v,m)] \pm uov \in Z(R)$, $[B_1(u,m),D_1(v,m)] \pm [u,v] \in Z(R)$, $[B_1(u,m),u] \pm [u,D_1(u,m)] \in Z(R)$ and $B_1(u,m)ou \pm uoD_1(u,m) \in Z(R)$, for every $u,v,m \in I$.

Keywords: commutative, prime ring, biderivation, ideal, and derivation.

1. INTRODUCTION

In recent years, several authors have investigated the concept of commutativity of prime rings by considering derivations (see [2],[3],[8],[11]and [12]), as well as commutativity of prime rings by considering generalized derivations (see [4],[9] and [10]).The concept of derivations with prime rings was initiated by E.C. Posner[1] in the year 1957. H.E. Bell and S. Martindale III[2] exposed that a semiprime ring R must have a nontrivial central ideal if it admits an appropriate endomorphism or derivation which centralizing on some nontrival one sided ideal. I.N. Herstein[5] determined the structure of a prime ring R which has a derivation $d \neq 0$ such that the values of d commute i.e for which $d(x)d(y) = d(y)d(x)$, for any $x,y \in R$. Shakir Ali and H Shuliang[14] produced the commutativity results for rings and presented that if R is a 2-torsion free semiprime ring and I a nonzero ideal of R, then a derivation d of R is commuting on I if one of the following conditions holds i. $d(x)od(y) = xoy$, ii. $d(x)od(y) = -(xoy)$, iii. $d(x)od(y) = 0$, iv. $[d(x),d(y)] = -[x,y]$, v. $d(x)d(y) = xy$ and vi. $d(x)d(y) = -(xy)$, for any $x,y \in I$. M. Ashraf and N.Ur. Rehman [12] shown that R is commutative $\Leftrightarrow$ it satisfies any one of the properties $d(xy) \pm xy \in Z(R)$, $d(xy) \pm yx \in Z(R)$ and $d(x)d(y) \pm xy \in Z(R)$, for any $x,y \in I$. M.A. Quadri, M. Shadab Khan and N. Rehman[9] extended these results to generalized derivations and also in recent days M.K.AbuNawas and RadwanM.Al-Omary [13] contuined the above results of commutativity of prime rings with commutator and anticommutator identities see theorem 3.1-theorem 3.8 [13].

The concept of symmetricbiderivations on prime and semiprime rings was introduced by J.Vukman[6].The notation and terminology in this paper follows [6][7]. Now in this present paper we continued the results of M.K.AbuNawas and RadwanM.Al-Omary [13] using symmetric biderivations, i.e. a prime ring R with nonzero left ideal I of R, which satisfies certain conditions, namely $B_1(u,m)ou \in Z(R)$, $[B_1(u,m),B_1(v,m)] - B_1([u,v],m) \in Z(R)$,$B_1(u,m)oB_1(v,m) - B_1(uov.m) \in Z(R)$,$B_1([u,v],m) + [B_1(u,m),y] - [B_1(u,m),B_1(v,m)] \in Z(R)$, $B_1(uov,m) \pm [u,v] \in Z(R)$, $[B_1(u,m),B_1(v,m)] \pm uov \in Z(R)$, $[B_1(u,m),D_1(v,m)] \pm [u,v] \in Z(R)$, $[B_1(u,m),u] \pm [u,D_1(u,m)] \in Z(R)$ and $B_1(u,m)ou \pm uoD_1(u,m) \in Z(R)$, for every $u,v,m \in I$., then R is commutative.

2. PRELIMINARIES

In each part of this article all rings assumed to be associative and possesses an identity with center of a ring R is$Z(R)$. As a wellknownfor every $x,y \in R$, the commutator $(xy - yx)$ be denoted by $[x,y]$ and the anticommutator $(xy + yx)$ be denoted by xoy. Recall that R is a prime ring if $xRy = 0$ implies $x = 0$ or $y = 0$ and is a semiprime if $xRx = 0$implies $x = 0$. An additive map $d: R \to R$ is called derivation if $d(xy) = d(x)y + xd(y)$, for all $x,y \in R$. A biadditive mapping $B(.,.): R \times R \to R$ is called symmetric if $B_1(x,y) = B_1(y,x)$, for any $x,y \in R$. A biadditive mapping $B_1(.,.): R \times R \to R$ is called a symmetricbiderivationif $B_1(xy,z) = B_1(x,z)y + xB_1(y,z)$, for all $x,y,z \in R$. Obviously, in this case also if $B_1(x,yz) = B_1(x,y)z + yB_1(x,z)$, for all $x,y,z \in R$. We use without mention, the following are basic commutator and anticommutator identities

$[xy,z] = x[y,z] + [x,z]y$, $[x,yz] = [x,y]z + y[x,z]$,
$xo(yz) = (xoy)z - y[x,z] = y(xoz) + [x,y]z$,

$(xy)oz = x(yoz) - [x,z]y = (xoz)y + x[y,z]$.

The following results are very significant to prove main theorems.

Remark: For a prime ring R, a nonzero element $a \in Z(R)$, if $ab \in Z(R)$, then $b \in Z(R)$.

Letus begin our discussion with the following lemmas

Lemma 2.1: For a prime ring R, if $B_1: R \times R \to R$ is a symmetric biderivation on R, then for any $w \neq 0 \in Z(R)$, $B_1(w,m) \in Z(R)$.

Proof: for any $w \neq 0 \in Z(R)$ which indicates$[w,r] = 0$, for any $r \in R$ and hence we write$B_1([w,r],m) = 0$, for any $m \in I$, then using commutator identity to find that$B_1(w,m)r + wB_1(r,m) - B_1(r,m)w - rB_1(w,m) = 0$. Since $w \in Z(R)$, $zB_1(r,m) = B_1(r,m)w$, therefore $B_1(w,m)r - rB_1(w,m) = 0$, which implies $[B_1(w,m),r] = 0$. From this it is clear that $B_1(w,m) \in Z(R)$.

Lemma 2.2: Let I be a nonzero left ideal of a prime ring R and if B_1 is a symmetric biderivation of R which is centralizing on I then B_1 is commuting on I.

Proof : For any $u \in I$, $[x, B_1(u,m)] \in Z(R)$, also $[u^2, B_1(u^2,m)] \in Z(R)$, on simplifying, we get $[u^2, uB_1(u,m) + B_1(u,m)u] \in Z(R)$, we shall rewritten it as $[u^2, 2uB_1(u,m) - [u, B_1(u,m)] \in Z(R)$ then it reduces to $4[u^2, B_1(u,m)][u, B_1(u,m)] = 0$ and $8u[u, B_1(u,m)]^2 = 0$. Hence $[u, B_1(u,m)]^3 = 0$ (1)

Using the property (ii) in [2], the center contains no nonzero nipotent elements. Therefore $2[u, B_1(u,m)] = 0$ and it follows that $[u^2, B_1(u,m)] = 0$. (2)

By linearizing both Eq. (1) and our original hypothesis, we see that

$[u, B_1(v,m)] + [v, B_1(u,m)] \in Z(R)$and

$2([u, B_1(v,m)] + [v, B_1(u,m)]) = 0$and combining these results with Eq. (1), we easily see that $[uv + vu, B_1(u,m)] + [u^2, B(v,m)] = 0$. (3)

Changing$v$withvu, we get

$$[uvu + vuu, B_1(u,m)] + [u^2, B_1(vu,m)] = 0$$

$(uv + vu)[u, B_1(u,m)] + [u^2, v]B_1(u,m) + v[u^2, B_1(u,m)] = 0$,rewriting the first summand as $([u,v] + 2vu)[u, B_1(u,m)]$ and using (1), (2) and (3), we find that $[u,v][u, B_1(u,m)] + [u^2, y]B_1(u,m) = 0$, use $v = B_1(u,m)u$ and by (2), we find that $[u, B_1(u,m)]u[u, B_1(u,m)] + [u^2, B_1(u,m)]uB_1(u,m) = 0$, thus we conclude $[u, B_1(u,m)] = 0$.

Using the same tricks as used here to prove the below lemma.

Lemma 2.3: For a prime ring R and a nonzero left ideal I of R, if R admits a nonzero symmetric biderivationB_1 such that $[u, B_1(u,m)] \in Z(R)$, for all $u, m \in I$ then R is commutative.

Lemma 2.4: For a prime ring R, contains a commutative nonzero right ideal, then R is commutative.

Lemma 2.5([13], lemma 2.5)**:**For a prime ring R and a nonzero left ideal I of R which satisfies one of the following condition

I. $[u,v] \in Z(R)$ or

II. $uov \in Z(R)$, for any $u, v \in I$. Then R is commutative.

3.MAINRESULTS

Theorem 3.1: For a prime ring R and a nonzero left ideal $I$of R, suppose that R admits a symmetric biderivationB_1 such that $B_1(Z(R),m) \neq 0$ further, if it satisfies the condition $B_1(u,m)ou \in Z(R)$, for any $u, m \in I$, then R is commutative.

Proof:consider $B_1(u,m)ou \in Z(R)$, for any $u, m \in I$.

Substituting$u$ by u+v, then $B_1(u+v,m)ou \in Z(R)$,we obtain

$$B_1(u,m)ov + B_1(v,m)ou \in Z(R). \quad (4)$$

Since $B_1(Z(R),m) \neq 0$, there exist $w \in Z(R)$ such that $B_1(w,m) \neq 0$. Replacing v by wv in (4) and using (4), then $B_1(u,m)owv + B_1(wv,m)ou \in Z(R)$, we attain$B_1(w,m)(vou) - [B_1(w,m),u]v \in Z(R)$. Therefore using lemma 2.1, $B_1(w,m) \neq 0 \in Z(R)$ hence we get $B_1(w,m)(vou) \in Z(R)$. Since R is prime ring and using remark we find that $vou \in Z(R)$ and hence by lemma 2.5(II), R is commutative.

Theorem 3.2: For a prime ring R and a nonzero left ideal $I$of R, suppose that if R admits a symmetric biderivationB_1 such that $B_1(Z(R),m) \neq 0$. Further if R satisfies any one of the following conditions:

i. $[B_1(u,m), B_1(v,m)] - B_1([u,v],m) \in Z(R)$

ii. $B_1(u,m)oB_1(v,m) - B_1(uov,m) \in Z(R)$, for any $u, v, m \in I$. Then R is commutative.

Proof: (i) consider $[B_1(u,m), B_1(v,m)] - B_1([u,v],m) \in Z(R)$ (5)

Since $B_1(Z(R),m) \neq 0$, there exist $w \in Z(R)$ such that $B_1(w,m) \neq 0$. Substituting$v$ by wv in (5) and byEq.(5), we get the expression

$[B_1(u,m),w]B_1(v,m)+w[B_1(u,m),B_1(v,m)]+[B_1(u,m),B_1(w,m)]v+B_1(w,m)[B_1(u,m),v]-[u,w]B_1(v,m)-B_1([u,w],m)v-\ wB_1([u,v],m)-B_1(w,m)[u,v]\in Z(R)$, since w is in center which commute with every element, then which reduces to

$$w[B_1(u,m),B_1(w,m)]v+B_1(w,m)[B_1(u,m),v]-B_1(w,m)[u,v]\in Z(R)$$

Using lemma 2.1, $B_1(w,m)\neq 0\in Z(R)$, so we get$B_1(w,m)([B_1(u,m),v]-[u,v])\in Z(R)$.
Since $B_1(w,m)\in Z(R)$, using remark, we find $[B_1(u,m),v]-[u,v]\in Z(R)$.
Replace v by $B_1(w,m)u$ in the above expression and use above expression to get

$$[B_1(u,m),B_1(w,m)u]-[u,B_1(w,m)u]\in Z(R)$$
$$[B_1(u,m),B_1(w,m)]u+B_1(w,m)[B_1(u,m),u]-[u,B_1(w,m)]u\in Z(R)$$

On simplifying, we have $B_1(w,m)[B_1(u,m),u]\in Z(R)$ using primeness of R, remark we get $[B_1(u,m),u]\in Z(R)$, then using Lemma 2.3, we get R is commutative.

(ii) consider$B_1(u,m)oB_1(v,m)-B_1(uov,m)\in Z(R)$ (6)
Since $B_1(Z(R),m)\neq 0$, there exist $w\in Z(R)$ such that $B_1(w,m)\neq 0$
Replacing v by wv in the above Eq.(6) and using the Eq.(6), we get
$B_1(u,m)oB_1(wv,m)-B_1(uowv,m)\in Z(R)$then we see the relation obtained

$$B_1(w,m)(B_1(u,m)ov)+[B_1(u,m),B_1(w,m)]v+w\big(B_1(u,m)oB_1(v,m)\big)+[B_1(u,m),w]B_1(v,m)-B_1(w,m)(uov)-wB_1(uov,m)-[u,w]B_1(v,m)-B_1([u,w],m)v\in Z(R)$$

Which gives $B_1(w,m)(B(u,m)ov)+[B_1(u,m),B_1(w,m)]v-B_1(w,m)(uov)\in Z(R)$.
Using lemma 2.1, $B_1(w,m)\in Z(R)$ then above expression becomes $B_1(w,m)((B_1(u,m)ov)-(uov))\in Z(R)$, from the remark ,we have seen that $B_1(u,m)ov-uov\in Z(R)$, then replacing u by wu, to get

$$B_1(w,m)(uov)-[B_1(w,m),v]u+w(B_1(u,m)ov)-[w,v]B_1(u,m)-w(uov)+[w,v]u\in Z(R).$$

Using the similar arguments as used above , it yields to $B_1(w,m)(uov)\in Z(R)$ and thus we conclude $uov\in Z(R)$. So using lemma 2.5(II), it is clear that R is commutative.

Theorem 3.3: For a prime ring R and a nonzero left ideal I of R, suppose that R admits a symmetric biderivationB_1 such that $B_1(Z(R),m)\neq 0$, further if R satisfies the condition $B_1([u,v],m)+[B_1(u,m),v]-[B_1(u,m),B_1(v,m)]\in Z(R)$, for any $u,v,m\in I$, then R is commutative.

Proof: for any $u,v,m\in I$, we have
$B_1([u,v],m)+[B_1(u,m),v]-[B_1(u,m),B_1(v,m)]\in Z(R)$. (7)
Since $B_1(Z(R),m)\neq 0$, there exist $w\in Z(R)$ such that $B_1(w,m)\neq 0$
Replacing v by wv in the above Eq.(7) and using the Eq.(7), we get
$B_1(w[u,v],m)+B_1([u,w]v,m)+[B_1(u,m),w]v+w[B_1(u,m),v]-[B_1(u,m),B_1(w,m)v+wB_1(v,m)]\in Z(R)$, then simplifying to get$B_1(w,m)([u,v]-[B_1(u,m),v])\in Z(R)$.
Since using lemma 2.1, $B_1(w,m)\in Z(R)$ and using remark that $[u,v]-[B_1(u,m),v]\in Z(R)$.
Substituting$v$by $B_1(w,m)u$, we obtain $[u,B_1(w,m)]u-[B_1(u,m),B_1(w,m)]u-B_1(w,m)[B_1(u,m),u]\in Z(R)$, which on simplification, we find $B_1(w,m)[B_1(u,m),u]\in Z(R)$,
with the use of $B_1(w,m)\in Z(R)$ and remark, we get $[B_1(u,m),x]\in Z(R)$, from the lemma 2.3 we conclude that$R$ is commutative.

Theorem 3.4: For a prime ring R and a nonzero left ideal $I$of R, suppose that R admits a symmetric biderivationB_1 such that $B_1(Z(R),m)\neq 0$, further if R satisfies one of the following condition

i. $B_1(uov,m)-[u,v]\in Z(R)$
ii. $B_1(uov,m)+[u,v]\in Z(R)$, for any $u,v,m\in I$, then R is commutative.

Proof: **(i)** consider $B_1(uov,m)-[u,v]\in Z(R)$
If $B_1=0$ then $[u,v]\in Z(R)$ and hence by lemma 2.5(I), we get the required result.
Therefore we shall assume that $B_1\neq 0$, then we have $B_1(uov,m)-[u,v]\in Z(R)$ (8)
Since $B_1(Z(R),m)\neq 0$, there exist $w\in Z(R)$ such that $B_1(w,m)\neq 0$.
Replacing v by wv in the above Eq.(8) and using the Eq.(8), we get
$B_1(w,m)(uov)+wB_1(uov,m)+B_1([u,w],m)v+[u,w]B_1(v,m)-[u,w]v-w[u,v]\in Z(R)$.
Which gives,$B_1(w,m)(uov)\in Z(R)$ and since $B_1(z,m)\in Z(R)$ by lemma 2.1 and use of remark, we get $(uov)\in Z(R)$, then we use the lemma 2.5(II) to get the required result.

(ii) proof of this condition follows using the similar arguments as done in (i).

Theorem 3.5: For a prime ring R and a nonzero left ideal $I$of R, suppose that R admits a symmetric biderivationB_1 such that $B_1(Z(R),m)\neq 0$, further if R satisfies one of the following condition

i. $[B_1(u,m),B_1(v,m)]-uov\in Z(R)$
ii. $[B_1(u,m),B_1(v,m)]+uov\in Z(R)$, for every $u,v,m\in I$, then R is commutative.

Proof: **(i)** we have $[B_1(u,m),B_1(v,m)]-uov\in Z(R)$
If $B_1=0$ then $uov\in Z(R)$ and hence by lemma 2.5(II), we get the required result.
Therefore we assume that $B_1\neq 0$, for every $u,v,m\in I$
Consider $[B_1(u,m),B_1(v,m)]-uov\in Z(R)$ (9)

Since $B_1(Z(R), m) \neq 0$, there exist $w \in Z(R)$ such that $B_1(w, m) \neq 0$.
Substituting v by wv in the above Eq.(9) and using the Eq.(9), we get
$B_1(w, m)[B_1(u, m), v] + [B_1(u, m), B_1(w, m)]v + [B_1(u, m), w]B_1(v, m) + w[B_1(u, m), B_1(v, m)] - w(uov) + [u, w]v \in Z(R)$, using the relation w is the element in center of R,

$$B_1(w, m)[B_1(u, m), v] + [B_1(u, m), B_1(w, m)]v + w[B_1(u, m), B_1(v, m)] - w(uov) \in Z(R)$$

Since $B_1(w, m) \in Z(R)$ by lemma 2.1, the above expression becomes, $B_1(w, m)[B_1(u, m), v] \in Z(R)$ and using remark, we get $[B_1(u, m), v] \in Z(R)$, in particular we have $[B_1(u, m), u] \in Z(R)$ and we conclude that from lemma 2.3, R is commutative.

(ii) proof of (ii) follows by proceeding the same arguments as above (i).

Theorem 3.6: For a prime ring R and a nonzero left ideal I of R, suppose that R admits a symmetric biderivations B_1 and D_1 such that $B_1(Z(R), m) \neq 0$ and $D_1(Z(R), m) \neq 0$, if R satisfies one of the following conditions

i. $[B_1(u, m), D_1(v, m)] - [u, v] \in Z(R)$
ii. $[B_1(u, m), D_1(v, m)] + [u, v] \in Z(R)$, for every $u, v, m \in I$, then R is commutative.

Proof : (i) first assume that the condition $[B_1(u, m), D_1(v, m)] - [u, v] \in Z(R)$. If $B_1 = 0 (or\ D_1 = 0)$, then $[u, v] \in Z(R)$. Hence by lemma 2.5(I), R is commutative .

Next we assume that, for B_1 and D_1 are nonzero symmetric biderivations such that $[B_1(u, m), D_1(v, m)] - [u, v] \in Z(R)$. (10)

Substituting v by wv in the above Eq.(10) and using the Eq.(10), we get
$[B_1(u, m), w]D_1(v, m) + w[B_1(u, m), D_1(v, m)] + D_1(w, m)[B_1(u, m), v] + [B_1(u, m), D_1(w, m)]v - [u, w]v - w[u, v] \in Z(R)$, which implies on simplifying

$$[B_1(u, m), w]D(v, m) + D_1(w, m)[B_1(u, m), v] \in Z(R)$$

Since $B_1(w, m) \in Z(R)$ by lemma 2.1, the above expression becomes,
$D_1(w, m)[B_1(u, m), v] \in Z(R)$, since $D_1(w, m) \in Z(R)$ by lemma 2.1, hence use remark to conclude $[B_1(u, m), v] \in Z(R)$ and in particular $[B_1(u, m), u] \in Z(R)$. Therefore it is clear from lemma 2.3, R is commutative.

(ii) similar procedure as we done above to get the proof.

Theorem 3.7: For a prime ring R and a nonzero left ideal I of R, suppose that R admits a symmetric biderivations B_1 and D_1 such that $\{w \in Z(R) / B_1(w, m) = D_1(w, m) \neq 0\} \neq \emptyset$, then if R satisfies any one of the following conditions:

i. $[B_1(u, m), u] - [u, D_1(u, m)] \in Z(R)$
ii. $[B_1(u, m), u] + [u, D_1(u, m)] \in Z(R)$, for any $u, m \in I$, then R is commutative.

Proof:(i) firstassume that $[B_1(u, m), u] - [u, D_1(u, m)] \in Z(R)$

If $B_1 = 0 (or\ D_1 = 0)$, then $[u, D_1(u, m)] \in Z(R)$ (or $[B_1(u, m), u] \in Z(R)$), in both the cases using lemma 2.3, we get the required result.

So we assume that B_1 and D_1 are nonzerosymmetric biderivations, we have $[B_1(u, m), u] - [u, D_1(u, m)] \in Z(R)$, linearizing the above expression, we get

$$[B_1(u + v, m), u + v] - [u + v, D_1(u + v, m)] \in Z(R)$$

$[B_1(u, m), v] + [B_1(v, m), u] - [v, D_1(u, m)] - [u, D_1(v, m)] \in Z(R)$. (11)

Since $B_1(Z(R), m) \neq 0$, there exist $w \in Z(R)$ such that $B_1(w, m) \neq 0$, in the same manner $D_1(w, m) \neq 0$. Replacing v by wv in the above Eq.(11) and using the Eq.(11), we get
$w[B_1(u, m), v] + [B_1(u, m), w]v + [w, u]B_1(v, m) + w[B_1(v, m), u] + B_1(w, m)[v, u] + [B_1(w, m), u]v - w[v, D_1(u, m)] - [w, D_1(u, m)]v - D_1(w, m)[u, v] - [u, D_1(w, m)]v - w[u, D_1(v, m)] - [u, w]D_1(v, m) \in Z(R)$, which on simplifying, we get $B(w, m)[v, u] + [B_1(w, m), u]v - [u, D_1(w, m)]v - D_1(w, m)[u, v] \in Z(R)$. Use this relation $B_1(w, m) = D_1(w, m) \in Z(R)$ which is obtained from lemma 2.1, to get $B_1(w, m)[v, u] - D_1(w, m)[u, v] \in Z(R)$, which implies $B_1(w, m)[v, u] \in Z(R)$ or $D_1(w, m)[u, v] \in Z(R)$.
In both the cases using remark, $[u, v] \in Z(R)$ and hence using lemma 2.5(I), we conclude R is commutative.

(ii) similarlywe can prove (ii).

Theorem 3.8: For a prime ring R and a nonzero left ideal I of R, suppose that R admits a symmetric biderivations B_1 and D_1 such that $\{w \in Z(R) / B_1(w, m) = D_1(w, m) \neq 0\} \neq \emptyset$, then if R satisfies any one of the following conditions:

i. $B_1(u, m)ou - uoD_1(u, m) \in Z(R)$
ii. $B_1(u, m)ou + uoD_1(u, m) \in Z(R)$, for any $u, m \in I$, then R is commutative.

Proof: (i) firstassume that $B_1(u, m)ou - uoD_1(u, m) \in Z(R)$, if either of the symmetric biderivations is zero then either $B_1(u, m)ou \in Z(R)$ or $uoD_1(u, m) \in Z(R)$, hence in both the cases by theorem 3.1, we obtain our result.Now assume, for nonzero symmetric biderivations B_1 and D_1 such that $B_1(u, m)ou - uoD_1(u, m) \in Z(R)$. Linearizing the above equation , we find that $B_1(u, m)ov + B_1(v, m)ou - uoD_1(v, m) - voD_1(u, m) \in Z(R)$ (12)

Since $B_1(Z(R), m) \neq 0$, there exist $w \in Z(R)$ such that $B_1(w, m) \neq 0$, in the same manner $D_1(w, m) \neq 0$.Replacing v by wv in the above Eq.(12) and using the Eq.(12), we get
$B_1(w, m)(vou) - [B_1(w, m), u]v - D_1(w, m)(uov) - [u, D_1(w, m)]v \in Z(R)$.
Use this relation $B_1(w, m) = D_1(w, m) \in Z(R)$ which is obtained from lemma 2.1, to get
$B_1(w, m)(vou) - D_1(w, m)(uov) \in Z(R)$,which implies $B_1(w, m)(vou) \in Z(R)$ or $D_1(w, m)(uov) \in Z(R)$. In both the cases using remark, $(uov) \in Z(R)$ and hence using lemma 2.5(II), we conclude that R is commutative.
(ii) similarly we can prove (ii).

REFERENCES

[1] E.C. Posner, "Derivations in prime rings", Proc.Amer.Math.Soc.,Vol.8, (1957), pp.1093-1100.
[2] H.E. Bell and W.S. Martindale, "Centralizing mappings of semiprime rings", canad.Math. Bull.,Vol.30,(1987), pp.92-101.
[3] H.E. Bell and M.N. Daif, "On commutativity and strong commutativity preserving maps", canad.Math. Bull.,Vol.31, (1994), pp.443-447.
[4] H.E. Bell and N. Rahman, "Generalized derivations with commutativity and anticommutativity conditions", Math. J. Okayama Univ., Vol.49,(2007),pp.139-147.
[5] I.N. Herstein, "Rings with Involution". Uni.Chicagopres,. Chicago, (1976).
[6] J. Vukman, "Symmetric biderivations on prime and Semiprime rings",Aeq.Math., Vol.38, (1989),pp.245-254.
[7] J. Vukman, "Two results concerning symmetricbiderivations on prime rings". Aeq.Math.,Vol.40,(1990), pp.181-189.
[8] M. Bresar, "Centralizing mappings and derivations in prime rings", J.algebraVol.156. (1993), pp.385-394.
[9] M.A. Quadri, M. Shadab khan and N. Rahman, "Generalized derivations and commutativity of prime rings", Indian J. pure appl. Math., Vol.34. No.(9),(2003) pp.1393-1396.
[10] M. Ashraf, A. Ali and S. Ali, "Some Commutativity Theorems for rings with Generalized Derivations", Southeast Asian Bullitin of Mathematics,Vol.31,(2007),pp415-421.
[11] M. Ashraf and N.ur. Rehman, "on commutativity of rings with derivations", Results Math., Vol.42, (2002) pp.3-8.
[12] M. Ashraf and N.ur. Rehman, "On derivation and commutativity in Prime rings". East-West J. of Mathematics,Vol.3, No.(1),(2001),pp.87-91.
[13] M. K. Abu Nawasand Radwan M. AL-Omary, "On ideals and commutativity of prime rings with generalized derivations", European journal of pure and applied mathematics, Vol.11, No.(1),(2018),pp.79-89.
[14] S. Ali and H. Shuliang, "On deivations in Semiprime rings". Algebr Represent Theor., Vol.15,(2012),pp.1023-1033.