Prediction of Stochastic Spatial Processes Applied to the Water of Wells

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Abstract

The research includes the prediction of the spatial random process by two Techniques, namely the multiple linear regression technique and the universal Kriging Technique. The two techniques were applied to real data representing the height of water table in 28 wells with the coordinates of their locations. It was found that the Kriging Technique is better than the least squares Technique through the results and numerical values obtained depending on the standard of mean square error. Where, the prediction of random spatial processes known as the Kriging Technique gives good results, depending on the function of the Variogram in finding the predictions.

Key words: spatial processes, mean square error , prediction.

Introduction

The research dealt with the prediction of the random process by the regression technique, whereby the regression analysis deals with describing the spatial data present in a specific area or a specific subspace of Euclid's space that includes agricultural applications or the presence of minerals or groundwater. These phenomena are measured by a polynomial model in terms of the regression parameters and the coordinates of the spatial locations of the data. As well as the prediction of the spatial random process by the Kriging Technique (in spatial statistics), Baudin et al. (2017) a technique used to predict in particular about spatial phenomena such as minerals above and below ground , groundwater and environmental pollution as well as predicting the spread of epidemic diseases and natural disasters in economic fields(Diggle and Riberro(2007). Kriging Technique is also used in any study if it is possible to define the phenomenon under study on the basis of the distance between the data samples of this phenomenon, Dong and Nakayama (2017), and Le Guiban et al. (2017)).

Kriging Technique in prediction has reduced the use of the regression Technique in prediction, because the regression analysis Technique requires the definition of one variable or several explanatory variables, while Kriging one only includes knowledge of the distance between the observations of the phenomenon. Moreover, the mean square error of the modification in Kriging Technique is always less than regression. (See Sarma (2009), Calli (2010), Gaetan and Guyon (2010), Haining (2004).

This is what has been practically proved by applying the two Techniques to real data, representing the high levels of water table for 26 wells, with the coordinates of their locations in Sinjar district (Al-Hassoud 1985). The second section gives a description of the multiple linear regression and its estimated parameters using the least squares method . Whereas, the third section explains the random process and the variocram function and its characteristics. The fourth section handles predicting the spatial process depending on Kriging Technique, and finally the fifth section presents the practical aspect. see Erickson et al. (2017).

Research Aim

The research includes the prediction of the spatial random process by two Techniques, namely the multiple linear regression technique and the universal Kriging Technique and comparison of two methods terms of results.

Multiple linear regression

Most regression problems include more than one independent variable. In general, the multiple linear regression model can be written as follows:

$$z(x_i) = \beta_0 + \beta_1 x_{i1} + \cdots + \beta_k x_{ik} + e_i \qquad (1)$$

$$= \beta_0 + \sum_{j=1}^{k} \beta_j x_{ij} + e_i$$

$i = 1,2,\cdots,n$

$j = 1,2,\cdots,k$

Where $x_{ij}$is the view *i* of the independent variable *j*

$\beta_0, \beta_1, \beta_2, \cdots, \beta_k$ are unknown parameters that require their estimation from the available data.In matrix form, equation (1) is written as:

$$z = X\beta + e \qquad (2)$$

This model is called the multiple linear regression model, where z is a vector with a capacity of $n \times 1$ of observations, X is a known matrix with a capacity of $\left(n \times (k+1)\right)$ of observations and β is a vector of unknown parameters with capacity $(k+1)$ and that e is a random error vector with an $n \times 1$ dimension belonging to the normal distribution at rate *0* and variance $\sigma^2 I_n$.

That is:

$E(e) = 0$

$var(e) = \sigma^2 I_n$

I_n: is a one-dimensional matrix with dimension nxn.

Depending on the least squares Technique, the estimator of the parameters vector will be as follows (Draper and Smith (1981):

$$\hat{\beta} = (X^t X)^{-1} X^t Z \quad (3)$$

Whereas, $\beta^{\wedge}$ is unbiased estimator for β and has the characteristic of the best linear unbiased estimator (BLUE). Likewise:

$$\operatorname{var}(\hat{\beta}) = \sigma^2 (X^t X)^{-1} \quad (4)$$

Stochastic process

The spatial stochastic process is defined as a set of random variables $(T \in X_t)$ defined by the $t \geq 0$index on a continuous or intermittent potential space and has values in this space. Whereas the spatial random process is that its variables are spatial data such as the existence of minerals inside the ground studied by mining engineering and groundwater and estimating the reserves of various materials.

All of these phenomena fall within the concept of spatial or geological statistics, and the goal is usually to identify the structural body of the variables in the studied geographical area, and the correlation and relationships between them. The values of the spatial variables represent a real function that takes a specific value within the region or geographical field in a one-dimensional space with the point $z(u_1)$ or two dimensions with the point $z(u_1, u_2)$ or three dimensions with the point $z(u_1, u_2, u_3)$.The spatial linear model of the variable $z(x)$can be written in the field or region *D* which is a subset of the space R^P in the form of matrices as follows:

$$z(x) = f^t(x)\beta + \varepsilon(x) \ldots x \in D \subseteq R^P \quad (5)$$

where:

$$z = \left(z(x_1), z(x_2), z(x_3) \cdots, z(x_n)\right)^t$$

$$X = f^t(x) = \left(f_1(x_1), f_2(x_2), f_2(x_3), \cdots, f_n(x_n)\right)^t$$

$$\beta = (\beta_1, \beta_2, \beta_3, \cdots, \beta_k)^t$$

$$\varepsilon = \left(\varepsilon(x_1), \varepsilon(x_2), z(x_3), \cdots, \varepsilon(x_n)\right)$$

Among the hypotheses of the model are the following:

1 - $E\left(Z(x)\right) = f^t(x)\beta$

2 - cov $[Z(x+h), z(x)] \cdots,$ exists

3 - $E[Z(x+h) - Z(x)]^2 = 2\gamma(h) \quad , \ (x, x+h) \epsilon D \ \ldots.. \quad (6)$

The dependency or independence relationship between random process variables or spatial variables is measured by the Variogram function, and this function is considered an alternative measure of correlation because the variations between the observations of the values of spatial variables are sometimes large or unknown, so the resulting correlation coefficients are small, which makes the results and interpretations about them wrong or inaccurate, on the other hand, and in the event of instability, variation is rarely known, and for this the variogram function has been proposed with a displacement of h and the formula of the function, which represents the expected growth square between the values or locations as follows:

$$2\gamma(h) = E[Z(x+h) - Z(x)]^2$$

Based on the data under study, the Variogram function is estimated by the relation :

$$2\gamma(h) = \frac{1}{n}\sum_{i=1}^{n} E[Z(x_i + h) - Z(x_i)]^2 \qquad (7)$$

This function depends mainly on the distance *h* and not on the location *x*.
:As for $\gamma(h)$, it is called a semi variogram, and its main characteristics are
1 - It is positive $\gamma(h) \geq 0$ because of the quadratic formula and the distance at which the function that increases with increasing distance *h becomes stable* , and the height at this stability point is called the initial variation, Sill.

2 -It is isotropic on all sides angles (when stable) i.e. it depends on the distance h between the variable points only and not on the direction (Cressie (1993).

3. It is an even function in which $\gamma(h) = \gamma(-h)$ and Symmetric $\gamma(x, y) = \gamma(y, x)$) and its purpose at $h \to \infty$is given to $var\big(Z(x)\big)$ While the purpose of the Semi-variogram function is the so-called sill.

4 - For the purposes of prediction, a description or estimation is usually made for it with one of the proposed models in this field, the most famous of which is the spherical, natural, exponential, and linear model, among others.

5. The function has the phenomenon of Nugget or discontinuity. When it becomes h = *0*, the function is not equal to zero.

6. Variogram function is related to heterogeneity (common contrast) in the case that random operations are stable as follows:

$$2\gamma(h) = E[Z(x+h) - Z(x)]^2$$
$$= EZ^2(x) + EZ^2(x+h) - 2cov[Z(x+h).z(x)]$$
$$= var[Z(x)] + var[Z(x+h)] - 2C(h)$$
$$\gamma(h) = C(0) - C(h) \qquad (8)$$

Where C (h) represents the covariance function, and when offset h = 0, it is

$$var[Z(x)] = C(0)$$

Predicting the stochastic process

It is a prediction Technique used in the mining industry to predict input and estimate a mass model of mineral resources. There are several known Techniques for predicting spatial processes, the most important of which is the simple Kriging, Kleijnen and Van Beers (2017), Mukhopadhyay et al. (2016).. which is the weighted average of the values of $z(x)$ and the adjacent points around the location $z(x_0)$ to be predicted on the assumption that any point has some degree to do with $z(x_0)$ and this type is of little use which assumes that the model is stable and that the forecast is known or equal to zero $\left(\mu(x) = 0\right)$, and the spatial variance is known or not dependent on *x*.The second technique of prediction is the ordinary prediction. The ordinary prediction model for a stable, random, spatial process is what is known as the ordinary Kriging, which assumes the following:

$$Z(X) = \mu + \varepsilon(X) \qquad x \in D \qquad (9)$$

When μ is known, we will get the following formula prediction:

$$z^0(x) = \sum_{i=1}^{n} w_i Z(x_i)$$

Where: $\sum_{i=1}^{n} w_i = 1$ (10)

And thatω_i is the vector of weights, and the constraint $\sum_{i=1}^{n} w_i = 1$ *guarantees an unbiased prediction, that is:*

$$EZ^0(x) = Ez(x) = \mu$$

It is the estimated value of weights that makes the squared difference between predictive and original values as minimal as possible, or that minimizes the mean squared error of prediction by using the Lacrange technique:

$$var[z(x)] = E[Z(x) - z^0(x)]^2 \qquad (1\ 1)$$

We will get the following formula (Elias 2008), Minnitt and Assibey-Bonsu (2013).

$$W = \gamma^{-1}\left[\gamma_\circ + \left(\frac{1-\gamma_0^t \gamma l}{l^t \gamma^{-1} l}\right) l\right] \qquad (12)$$

where

l=(1,1,1,⋯,1)

$$\gamma_0 = (\gamma_{01}, \gamma_{02}, \cdots, \gamma_{0n})^t$$

As for the mini-prediction variance, which represents the Kirging variance, we obtain it from the compensation of the estimated values to w and the Lacrange coefficients estimates used in the prediction variance formula as in equation (11), Mehdad and Kleijnen (2017a).Another prediction is the universal prediction, or the so-called universal Kriging, that the universal prediction model pertains to the unstable random spatial process in which the average of spatial process is variable from one location to another and the model is written as follows:

$$z(X) = \mu(x) + \varepsilon(X) \qquad x \in D \qquad (13)$$

The best unbiased linear predictor $\hat{z}(x_0)$is as follows:

$$\hat{z}(x_0) = \sum_{i=1}^{n} W_i z(x_i) \qquad (14)$$

where

$$z = \left(z(x_1), z(x_2), \cdots, z(x_n)\right)^t \; W = \gamma^{-1}\gamma_0 - \gamma^{-1}X\left((X^t\gamma^{-1}X)^{-1}\left(X^t\gamma^{-1}\gamma_0 - f(x_0)\right)\right) \qquad (15)$$

For $\hat{z}(x_0)$ to be a better unbiased linear predictor, it must fulfill the following two conditions

1 - $E\left(\hat{Z}(x_0)\right) = Z(x_0)$

2 - $\sigma_k^2 = \mathrm{var}\left(\hat{z}(x_0) - z(x_0)\right)$ is min .

This variation is called the Kirging variance. Khudair (2011), Mehdad and Kleijnen (2017b)has the following formula:

$$\sigma_K^2 = \gamma^{-1}\gamma_0 - \left[f^t\left(\gamma_0^t\gamma^{-1}X - f^t(x_0)\right)(X^t\gamma^{-1}X)^{-1}\left(X^t\gamma^{-1}\gamma_0 - f(x_0)\right)\right] \qquad (16)$$

The practical aspect

In this aspect, the multiple linear regression model has been applied to real data representing the height of the levels of wells water table in the Sinjar District in Nineveh Governorate. These data represent (26) wells with their coordinates since $z(x_i)$represents the height of the water table per meter at the well at the site $x_i = (u_i, v_i)$which represent the well coordinates at this point. (Al-Hasoud (1985).

Table (1) shows the real data along with its coordinates.

Table (1)

Height of water table *Y* in meters and coordinates *v*, *u*

Of Sinjar in Nineveh

u_i	v_i	$Y(u_i, v_i)$	u_i	v_i	$Y(u_i, v_i)$
0.4	3.6	3.3	0.9	3.9	3.6
0.5	4	5.8	3	4.5	3.4
0.5	3.8	3.1	2.9	4.6	3.3
0.6	3.9	5.5	2.9	4.5	3.4
0.7	4	5.8	3.4	4.2	3.2
0.8	4	3.6	3.4	4.5	3.1
2	4.1	5.2	4	4.5	7.4
2.1	4.3	5.3	3.8	4.5	7.3
2.2	4.5	5.7	1.5	4	5.2
2.4	4.2	5.7	3	4.2	7.3
3.3	4.4	3.3	2.3	4.2	3.2

1.9	3.7	6.2	2.6	4.4	3.5
1.8	4.2	5.1	3.4	4.6	3.6
2.2	3.6	4.6	1.2	3.8	5.7

Multiple Linear Regression Model

The best regression model for this data is the third order regression model $p + q = 3$, as shown in equation (1).

Where the number of estimated model coefficients for the direction surface $\hat{Z}$ was ten.

By applying the *OLS* method to find the predictive model for Table 1 data, where the Minitab 16 program was used to find the predictive regression model and the model was as follows:

$\hat{z}$ = - 3348 - 939 u + 2779 v + 553 uv - 96.2 u^2 - 857 v^2 + 27.22 u^2v - 82.67 uv^2 - 3.03 u^3 + 81.72v^3

Where Table (2) below shows the values of the model coefficients and the values of P to determine the significance of these coefficients and their presence in the model from their absence. P values less than 5% indicate the significance of that variable and its survival in the model.

Table (2)

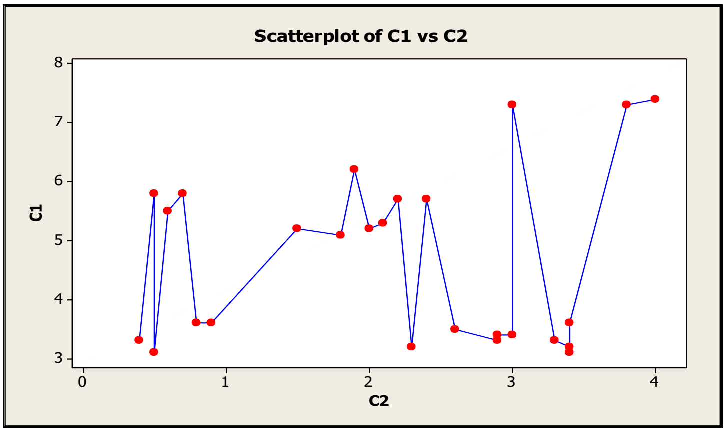
Values of Model coefficients and its tests

Predictor	Coef	SE Coef	T	P
Constant	-3348	2344	-1.39	0.185
u	-939.4	427.2	-2.18	0.045
v	2779	1912	1.51	0.152
uv	553.6	243.7	0.038	0.038
u^2	-96.17	37.05	0.021	0.021
v^2	-857.0	521.6	0.124	0.124
u^2v	27.22	10.54	0.024	0.024
uv^2	-82.67	34.67	0.033	0.033
u^3	-3.033	1.130	0.091	0.091
v^3	81.72	47.59	0.101	0.101

The value of the determination coefficient (R-Squared) $R^2 = 59.4\ \%$ which means that the model is $59.4\ \%$ data-appropriate and the mean square error value , MSE = 1.354 errors.

Kriging Technique Prediction

Before starting the approval of Kriging Technique, the data shape was drawn in Table (1). The data format can be represented as in Figure (1).



fig(1)

Where C1 represents $\hat{z}$ and C2 represents u .

For the purpose of knowing the suitability of the data for the model, smoothing technique was performed using the Median polish method, as in Figure (2), Figure (3).

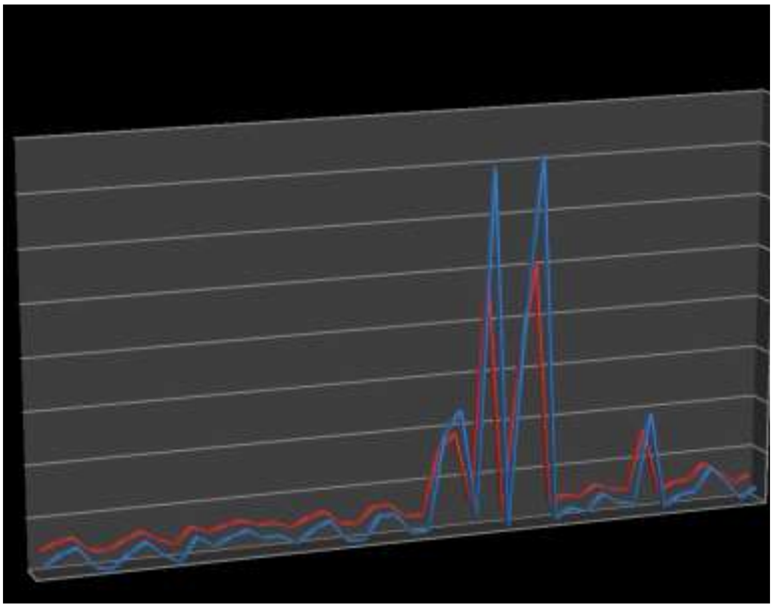
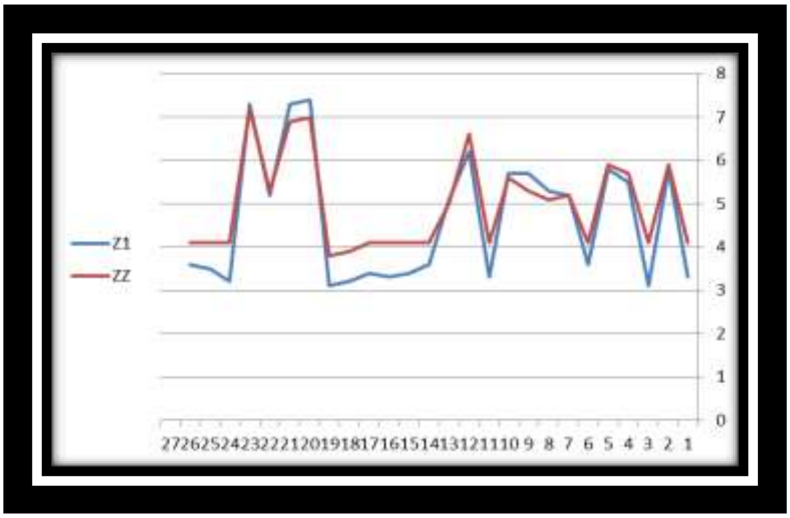


Figure (2)



Fig(3)

Values for the variable views z_1 and the values normalized by the median zz method

Data analysis and the suitability of the Variogram model:

The matrix of distances between (28) wells was calculated using the law of Euclidean Distance, which is denoted by the symbol h. See, (Sarma2009), (Haining2004).

$$h_{ij} = \sqrt{(u_i - u_j)^2 + (v_i - v_j)^2}$$

As each location is represented by coordinates:

(u_i, v_j) , i,j = 1, 2 ,….., 28

Thus, a matrix with a dimension (28 × 28) containing all distances between pairs of observations was obtained, and this distance was divided into ten categories, as these categories included the lowest value and the largest value of the distance, then calculating the center of each category to represent the approximate distance h. Likewise, the number of pairs of observations N(h) that are far from each other is calculated by the distance of h for each category center, and finally the index Variogram function was calculated for each category by applying equation (7) and the results are shown in Table (3)

Table (3)

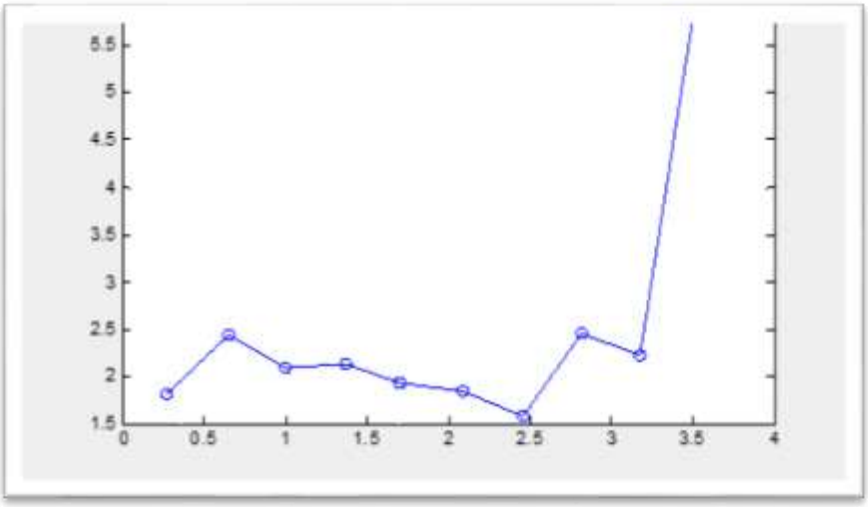
Approximate values of semi-Variogram function

categoies	minimum	maximum	Enter(h)	Pairs n(h)	$[Z(x_i)-Z(x_i+h)]^2$	$\gamma(h)$
Category(1)	0.2	0.543	0.2746	61	0.3119	1.808
Category(2)	0.4	0.802	0.6341	46	0.6110	2.4401

Category(3)	0.834	1.164	0.9975	44	0.9833	2.081
Category(4)	1.4	1.549	1.3658	47	1.3561	2.1402
Category(5)	1.64	1.853	1.6931	32	1.7226	1.934
Category(6)	1.93	2.266	2.0843	26	2.0759	1.843
Category(7)	2.28	2.640	2.4544	28	2.4750	1.585
Category(8)	2.648	2.973	2.8211	18	2.8144	2.454
Category(9)	3.019	3.347	3.1711	13	3.12110	2.218
Category(10)	3.374	3.720	3.5421	8	3.5474	6.256

By plotting the relationship between γ(h)versus *h* and then reconciling the semicircle curve, by passing the best curve between the points of the graph of the Semi-variogram function for the ten categories, we obtain Figure (4).

Fig(4)



We note that the curve of the Variogram pointed in Fig. 4 is closer to being similar to the spherical pattern

$$\gamma(h) = \begin{bmatrix} w_0 + w & , \quad h > a \\ w_0 + w\left(\frac{3h}{2a} - \frac{1}{2}(\frac{h}{a})^3\right) & , \quad h < a \\ w_0 & , \quad h = a \end{bmatrix}$$

As a= h is the range that represents the presence of the phenomenon under study, γ(h)= W_0 the value of the Nugget effect, which represents the weakness in the continuity and homogeneity of the phenomenon in the area under study, and $\omega_0 + \omega$ is the variance. Thus, this model will be used in the prediction process at non-measured locations.

As for the parameters of the indicative Variogram models (range, variance and nugget), they are as follows: We notice in Figure (3) that the curve of the indicative Variogram function crosses the vertical axis $\gamma(h)$at w_0=1.8070 and the curve increases ascending to settle at h = 3.541, and this determines the value of the range a = 3.543. We note that the Variogram function stabilizes at a height of 4.4520, which is approximately equal to the variance.

$\sigma^2 = w_0 + w$

Therefore w = 6.259

So, semi-Variogram function will be :

$$\gamma(\mathrm{h}) = \begin{bmatrix} 1.8070 + 4.4520 & , \quad \mathrm{h} > 3.5 \\ 1.8070 + 4.4520 \quad \left(\frac{3\mathrm{h}}{2(3.5)} - \frac{1}{2}\left(\frac{\mathrm{h}}{3.5}\right)^3\right) & , \quad \mathrm{h} < 3.5 \\ 1.8070 & , \quad \mathrm{h} = 3.5a \end{bmatrix}$$

Predictive values $\hat{z}(x_0)$ were found by (Matlab)

After that, we calculated the variation of Kriging for each site of the region under study and by taking the locations of the actual observations and their values in the study area and applying the random spatial process $\{z(x)\}$ that has the spherical model proposed above. For the purpose of applying equation (14) it is necessary to calculate the matrix of weights w This is done through equation (15) and this matrix has been calculated, as well as the variation of Kriging for each prediction according to equation (16). In the same way, other spatial locations are calculated or predicted, and table (4) shows the real values $z(x)$, predictive values $\hat{z}(x)$ and the kriging σ_k^2 variation values.

Table (4)
Prediction values of Kriging

No.	v	u	$z(x)$	w_i	$\hat{z}(x)$	σ_k^2
1	3.7	0.4	3.3	0.021645	3.2	1.1552
2	4.1	0.5	5.8	0.023198	4.7	1.3652
3	3.8	0.5	3.1	0.022782	3.4	2.1214
4	3.9	0.6	5.5	0.023914	4.8	1.2226
5	4.2	0.7	5.8	0.025206	5.2	1.0066
6	4	0.8	3.6	0.027106	3.8	1.4101
7	4.1	2.1	5.2	0.041607	4.3	0.9823
8	4.3	2.2	5.3	0.043255	4.8	1.0655
9	4.5	2.2	5.7	0.032164	5.6	1.0604
10	4.2	2.4	5.7	0.056083	5.6	1.8803
11	4.4	3.3	3.3	0.080713	2.2	2.54322
12	3.7	1.9	6.2	0.028572	5.7	1.0957

13	4.2	1.8	5.1	0.009461	5.5	1.3592
14	3.9	0.9	3.6	0.097307	3.2	1.8758
15	4.5	3.1	3.4	0.0500428	3.8	1.7646
16	4.6	2.9	3.3	0.049785	3.3	1.3652
17	4.5	2.9	3.4	0.050051	2.4	0.4061
18	4.2	3.4	3.2	0.049951	4.5	1.3652
19	4.5	3.4	3.1	0.050462	3.5	1.0165
20	4.5	4	7.4	0.047292	7.1	1.3643
21	4.5	3.8	7.3	0.048644	7.0	3.3953
22	4	1.5	5.2	0.035121	4.9	1.2251
23	4.2	3	7.3	0.049801	6.7	1.3636
24	4.2	2.2	3.2	0.045162	2.2	1.6074
25	4.4	2.6	3.5	0.048089	3.2	1.4236
26	4.6	3.4	3.6	0.022167	2.7	1.1063
27	3.6	2.2	4.6	0.049701	4.2	1.6774
28	3.8	1.2	5.7	0.089801	1	1.9074

Also, the mean square error has been calculated and MSE value = 0.0549. We note that the mean square error in Kriging technique is less than the mean square error in the regression technique by dropping the coefficients.

This is a clear evidence that Kriging technique is better than the regression technique, and on this basis we rely on the prediction of Kriging technique, and this represents a basic result in the reason for the prediction in the spatial statistics relying on Kriging technique.

Conclusions

Depending on the mean squared error, we note that Kriging technique is better than the multiple linear regression technique, since the value of the MSE is 0.0549, which is lower than it is in the case of relying on the least squares method, where the MSE value is 1.354.

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