

E-reverse super vertex magic labelings of graphs

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Abstract- Let a finite graph G with m vertices and n edges. A reverse vertex magic labelling is a bijection f from $V(G) \cup E(G)$ to the consecutive integers $1, 2, 3, \dots, m+n$ with the property that for every $u \in V(G)$, such that $f(u) - \sum_{v \in N(u)} f(uv) = k$ for some constant k . This type of labelling is E-reverse super magic if $f(E(G)) = \{1, 2, 3, \dots, q\}$. A graph G is called E-reverse super vertex magic if it admits a E-reverse super vertex magic labelling. Now in this paper, we explain some basic properties of such labelings and we establish E-reverse super vertex magic labelling of some families of graphs. We effort the focus of this paper is on the E-reverse super vertex magicness of $H_{m,n}$ and on some necessary conditions for a graph to be E-reverse super vertex magic.

Keywords – super vertex magic labeling, reverse super vertex magic labeling, Generalized Peterson graph, m -connected graph $H_{m,n}$.

I. INTRODUCTION

Let us consider the graph $G(V, E)$ be a simple undirected and finite with vertex set symbolically $V(G)$ and edge set symbolically $E(G)$, and let the number of vertices and edges of G be $p = |V(G)|$, $q = |E(G)|$, respectively. A graph $G = (V, E)$ is said to be a reverse vertex-magic labeling if there consists a constant k and a bijection $f : E \rightarrow \{p+1, p+2, \dots, p+q\}$ then we have the induced transformation $g_f : V \rightarrow N$, which is demarcated by $g_f(v) = k + \sum f(uv)$, is also injective and $g_f(V) = \{1, 2, \dots, p\}$. For this situation f is called a labelling of reverse vertex-magic of a graph G .

In [14,15], S.VenkataRamanaeta presented the idea of labelling of reverse super vertex magic of a graph. A labelling of reverse vertex-magic g is a bijection g from $V \cup E$ onto the numbers $\{1, 2, 3, \dots, v + \varepsilon\}$ so that for all vertex u , $g(N(u)) - g(u)$ is a constant. A labelling of reverse vertex-magic g is called labelling of reverse super vertex-magic if $f(E) = \{1, 2, 3, \dots, \varepsilon\}$ and $f(V) = \{\varepsilon + 1, \varepsilon + 2, \varepsilon + 3, \dots, \varepsilon + v\}$. A $G(V, E)$ graph is known as reverse super vertex-magic if there besides a labelling of reverse super vertex-magic of G .

The rest of the paper is organized as follows. Proposed embedding and extraction algorithms are explained in section II. Experimental results are presented in section III. Concluding remarks are given in section IV.

II. PROPOSED ALGORITHM

The following definition and results that will subsequently be very useful to prove some Theorems.

Definition 2.1:- The Corona $G_1 \circ G_2$ of two graphs G_1 and G_2 is formed by taking one copy of G_1 (which has p_1 vertices) and p_1 copies of G_2 and then joining the i th vertex of G_1 to every vertex in the i th copy of G_2 . C_n^+ is nothing but a corona $G_1 \circ G_2$ where $G_1 = C_n$ and $G_2 = K_1$.

Lemma 2.2:- If a non – trivial graph G is reverse super vertex magic , then the magic constant k is given by

$$k = q + \frac{p+1}{2} - \frac{q(q+1)}{p} .$$

Theorem 2.3:- Let G be a graph and f is a bijection from $E(G)$ onto $\{1,2,3,\dots,q\}$. Then f can be extended to a reverse super vertex magic labelling of G if and only if $\{w(u) = \sum_{v \in N(u)} f(uv) / u \in V(G)\}$ consists of p sequential integers.

III. EXPERIMENT AND RESULT

In this section, we prove some basic properties of E-reverse super vertex magic labelling. By applying the properties and the above lemma results which are showed in the previous section 1 , we prove the existence or non-existence of E-reverse super vertex magic labelling for some different families of graphs.

Theorem 3.1:- If G has a E-reverse super vertex magic with magic constant k , then $k \leq \frac{17p+3}{18}$

Proof:- This proof result follows directly from the above Lemmas 1.2 and 1.4.

By substituting Lemma 1.4 value in Lemma 1.2 then we can show as follows

$$\begin{aligned} k &= q + \frac{p+1}{2} - \frac{q(q+1)}{p} \\ &\leq \frac{2p}{3} + \frac{p+1}{2} - \frac{\frac{2p}{3}(\frac{2p}{3}+1)}{p} \\ &\leq \frac{12p+9p+9-4p-6}{18} \\ &\leq \frac{17p+3}{18} \end{aligned}$$

For verification, if n is odd and $n \geq 3$, then P_n is E-reverse super vertex magic labelling with magic constant

$$k = \frac{17p+3}{18} \quad (\text{see Figs.1 and 2}).$$

Theorem 3.2:- If the graph G is connected and also G has a E-reverse vertex magic constant k , then

$$k \geq \frac{p+1}{2} .$$

Proof:-If G is a connected graph then we have $q \geq p-1$. By Lemma 1.2

$$\begin{aligned} k &= q + \frac{p+1}{2} - \frac{q(q+1)}{p} \\ &\geq p-1 + \frac{p+1}{2} - \frac{(p-1)(p-1+1)}{p} \\ &= \frac{p+1}{2} . \end{aligned}$$

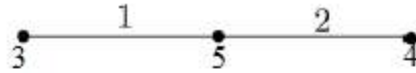


Fig. 1. $P_3 : p = 3, q = 2$ and $k = 2$.

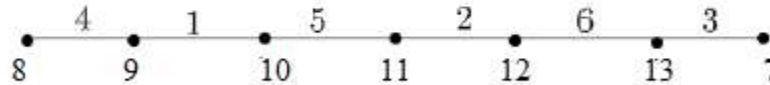


Fig. 2. $P_7 : p = 7, q = 6$ and $k = 4$.

Theorem 3.3:- Let the graph G be a (p,q) graph. If p is even and $q=p-1$ or p , then G is not E-reverse super vertex magic.

Proof:- Suppose there exists a E-reverse super vertex magic labelling of the graph G with magic constant k .

By Lemma 1.2, $k = q + \frac{p+1}{2} - \frac{q(q+1)}{p}$

Suppose at first $q = p-1$ then

$$k = p - 1 + \frac{p+1}{2} - \frac{(p-1)(p-1+1)}{p}$$

$$= \frac{p+1}{2} \text{ which is not an integer.}$$

Suppose next we take $q=p$, then by Lemma 1.2, we have

$$k = q + \frac{p+1}{2} - \frac{q(q+1)}{p}$$

$$= p + \frac{p+1}{2} - \frac{p(p+1)}{p}$$

$$= \frac{p-1}{2} \text{ which is also not an integer.}$$

Hence G is not E-reverse super vertex magic.

Corollary 3.4:- If p is even, then every tree T is not E-reverse super vertex magic.

Corollary 3.5:- C_n^+ (C_n sun graph) is not E-reverse super vertex magic.

Proof:- In C_n^+ , $p=2n$ and $q=2n$, By the above Theorem, C_n^+ is not E-reverse super vertex magic.

Theorem 3.6:- The generalized Peterson graph $P(n,m)$ is not E-reverse super vertex magic if n is odd.

Proof:- The Generalized Peterson graph $P(n,m)$ exists a E-reverse super vertex magic labelling with magic constant k . By Lemma 1.2, take $p=2n$ and $q=3n$

$$k = q + \frac{p+1}{2} - \frac{q(q+1)}{p}$$

$$= 3n + \frac{2n+1}{2} - \frac{3n(3n+1)}{2n}$$

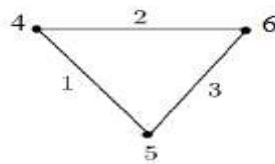
$$= \frac{-n-2}{2} \text{ which is not an integer if } n \text{ is odd.}$$

Theorem 3.7:- The fan graph F_n is E-reverse super vertex magic if and only if $n=2$.

Proof:- The fan graph F_n exists a E-reverse super vertex magic labelling with magic constant k . Then by the Lemma 1.2, we have

$$\begin{aligned}
 k &= q + \frac{p+1}{2} - \frac{q(q+1)}{p} \\
 &= 2n-1 + \frac{n+2}{2} - \frac{(2n-1)2n}{n+1} \\
 &= \frac{-3n(n-3)}{2(n+1)}
 \end{aligned}$$

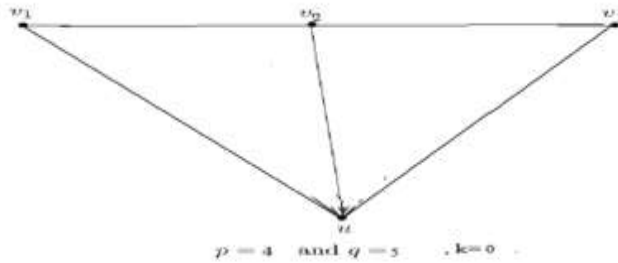
Which is not an integer for all n except 2,3 and 11.
 For the fan graph F_2 ,



$$p = 3, q = 3 \text{ and } k = 1$$

Fig. 3: Fan Graph F_2

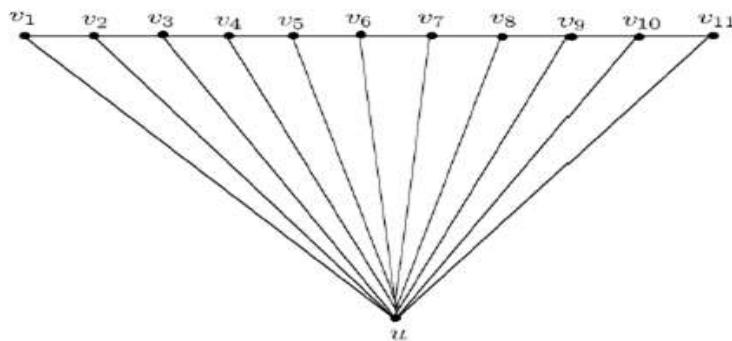
For the fan graph F_3 ,



$$p = 4 \text{ and } q = 5, k = 0$$

Fig. 4: Fan Graph F_3

For fan graph F_{11} ,



$$p = 12 \text{ and } q = 21, k = -11$$

Fig. 5: Fan Graph F_{11}

Suppose for the fan graphs F_3 and F_{11} with magic constants 0 and -11 by the lemma 1.2, but the edges are incident with u receives the minimum label 1 to 11.

3.8. E-reverse super vertex magicness of $H_{m,n}$

Here in this section we discuss the E-reverse super vertex magicness of m -connected graph $H_{m,n}$ on n vertices.

In [1] Bondy and Murty defined an m -connected graph $H_{m,n}$ on n vertices which has exactly $\lfloor mn/2 \rfloor$ edges. There are three cases on construction of structure of $H_{m,n}$ depends on the parties of m and n .

Theorem 3.8.1:- The m -connected graph $H_{m,n}$ is not E-reverse super vertex magic if both m and n are even.

Proof:- Suppose there exists a E-reverse super vertex magic labelling of $H_{m,n}$ with magic constant k . Then by using Lemma 1.2,

$$\begin{aligned} k &= q + \frac{p+1}{2} - \frac{q(q+1)}{p} \\ &= \frac{mn}{2} + \frac{n+1}{2} - \frac{\frac{mn}{2}(\frac{mn}{2}+1)}{n} \\ &= \frac{-m^2n + 2m(n-1) + 2n + 2}{4} \end{aligned}$$

Since m is even, $m=2r$ from case 1, therefore

$$\begin{aligned} k &= \frac{-4r^2n + 4r(n-1) + 2n + 2}{4} \\ &= \frac{-2r^2n + 2r(n-1) + n + 1}{2} \\ &= -r^2n + rn - r + \frac{n+1}{2} \end{aligned}$$

If n is even, $\frac{n+1}{2}$ is not an integer. So the magic constant k is not an integer for all m and n even. Therefore $H_{m,n}$

is not E-reverse super vertex magic.

Theorem 3.8.2:- $H_{m,n}$ is not E-reverse super vertex magic if m is odd and 4 does not divides n .

Proof:- By using Lemma 1.2 and using the above Theorem 3.1

$$k = \frac{-m^2n + 2m(n-1) + 2n + 2}{4}$$

If m is odd, let $m=2r-1$, then

$$\begin{aligned} k &= \frac{-(2r-1)^2n + (2r-1)(2n-2) + 2n + 2}{4} \\ &= \frac{-4r^2n + 4rn - n + 4rn - 4r + 4}{4} \\ &= -r^2n + 2rn - r + 1 - \frac{n}{4} \text{ which is not an integer.} \end{aligned}$$

Thus $H_{m,n}$ is not E-reverse super vertex magic.

Corollary 3.8.3:- $H_{m,n}$ is not E-reverse super vertex magic if both m and n are odd.

Theorem 3.8.4:- $H_{4,n}$ is E-reverse super vertex magic if and only if n is odd.

Proof:- The magic constant k of the graph $H_{4,n}$'s E-reverse super vertex magic labelling and by Theorem 3.1, $H_{4,n}$ is not E-reverse super vertex magic if n is even.

By Lemma 1.2, we have if n is an odd integer

$$\begin{aligned}
 k &= q + \frac{p+1}{2} - \frac{q(q+1)}{p} \\
 &= \frac{4n}{2} + \frac{n+1}{2} - \frac{\frac{4n}{2}(\frac{4n}{2}+1)}{n} \\
 &= 2n + \frac{n+1}{2} - 2(2n+1) \\
 &= -\frac{3}{2}(n+1)
 \end{aligned}$$

In the other hand, Let $V = \{v_1, v_2, v_3, \dots, v_n\}$ be the vertex set of $H_{4,n}$ and $E = \{v_i v_{i+1} / 1 \leq i \leq n-1\} \cup \{v_n v_1\} \cup \{v_i v_{i+2} / 1 \leq i \leq n-2\} \cup \{v_{n-1} v_1\} \cup \{v_n v_2\}$.

Define $f : V \cup E \rightarrow \{1, 2, 3, \dots, 3n\}$ as follows

$$f(v_i) = \begin{cases} \frac{1}{2}(4n+1+i) & \text{for } i \equiv 1(\text{mod } 2) \\ \frac{1}{2}(5n+1+i) & \text{for } i \equiv 0(\text{mod } 2) \end{cases}$$

$$f(v_n v_1) = 1$$

$$f(v_i v_{i+1}) = \begin{cases} \frac{1}{2}(n+2+i) & \text{for } i \equiv 1(\text{mod } 2), i \neq n \\ \frac{1}{2}(i+2) & \text{for } i \equiv 0(\text{mod } 2) \end{cases}$$

$$f(v_i v_{i+2}) = \begin{cases} 2(n-i) & \text{for } i = 1, 2, \dots, \frac{n-1}{2} \\ 3n-2i & \text{for } i = \frac{n+1}{2}, \frac{n+3}{2}, \dots, n-2. \end{cases}$$

$$f(v_{n-1} v_1) = n+2$$

$$f(v_n v_2) = 2n$$

It is easily seen that f is a E-reverse super vertex magic labelling with the magic constant $k = -\frac{3}{2}(n+1)$.

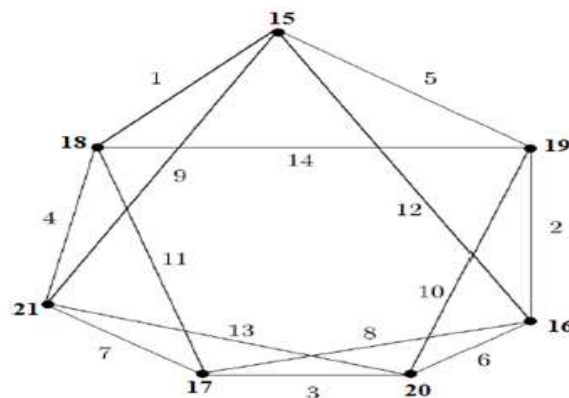


Fig. 6. $H_{4,7}$: $p = 7, q = 14$ and $k = -12$

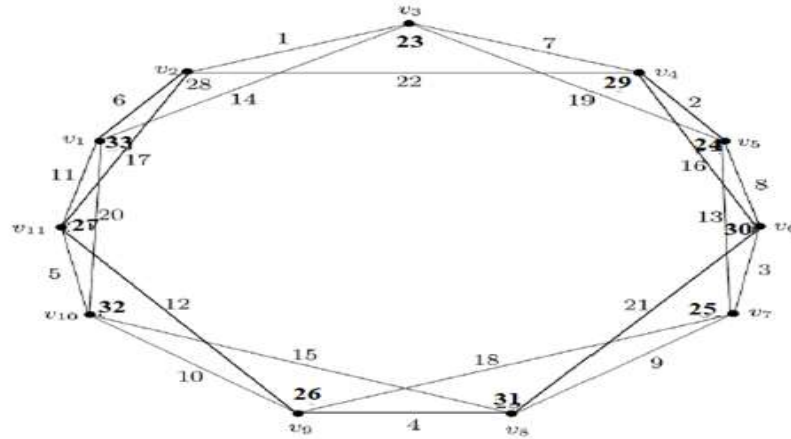


Fig. 7. $H_{4,11}$, $p = 11$, $q = 22$ and $k = 18$

Note that the graph $H_{4,7}$ can be decomposed into two Hamilton cycles. The assigned labels in Fig6 on the edges of one of the Hamilton cycles are 4,1,5,2,6,3,7. On the other Hamilton cycle edges are labelled by 11,8,12,9,13,10,14.

Theorem 3.8.5:- Let G be an odd order (p,q) graph and be decomposed into two Hamilton cycles, then G is E-reverse super vertex magic.

Proof:-Let G be a graph with p vertices and the odd length of the Hamilton cycle say $p=2m+1$. The graph G has the number of edges is $q=4m+2$. Then we label the Hamilton cycles with the following manner.

For the first Hamilton cycle label the edges which is starting at vertex v_1 with clockwise direction by

$$\frac{p+1}{2}, 1, \frac{p+3}{2}, 2, \frac{p+5}{2}, \dots, \frac{p-1}{2}, p.$$

For the second Hamilton cycle label the edges which is starting at vertex v_2 with clockwise direction by

$$\frac{3p+1}{2}, p+1, \frac{3p+3}{2}, p+2, \frac{3p+5}{2}, \dots, \frac{3p-1}{2}, 2p.$$

The sum of the edge labels at all the vertices are of p consecutive integers by direct inspection. So by Theorem 1.3 the result follows.

Theorem 3.8.6:- If a graph G can be decomposed into two spanning subgraphs G_1 and G_2 is E-reverse super vertex magic and G_2 is magic and regular, then G is E-reverse super vertex magic.

Proof:- Let G_1 and G_2 be any two graphs with q_1 and q_2 edges respectively. Let f_1 and f_2 denote the E-reverse super vertex magic labelling of G_1 and G_2 respectively.

In the following way, we label the edges as

Define $f : E(G) \rightarrow \{1, 2, \dots, q_1 + q_2\}$ where $q = q_1 + q_2$ then by edge label is given by

$$f(e) = \begin{cases} f_1(e) & \text{if } e \in E(G_1) \\ f_1(e) + q_1 & \text{if } e \in E(G_2) \end{cases}$$

Then for any vertex $u \in V(G)$,

$$\begin{aligned} w(u) &= \sum_{uv \in E(G)} f(uv) \\ &= \sum_{uv \in E(G_1)} f(uv) + \sum_{uv \in E(G_2)} f(uv) \\ &= \sum_{uv \in E(G_1)} f_1(uv) + \sum_{uv \in E(G_2)} (f_2(uv) + q_1) \\ &= \sum_{uv \in E(G_1)} f_1(uv) + \sum_{uv \in E(G_2)} f_2(uv) + rq_1, \text{ since } G_2 \text{ is } r\text{-regular.} \end{aligned}$$

Thus G_1 is E-reverse super vertex magic and by Theorem 1.3, $\{w(u)/u \in V(G_1)\}$ consists of p consecutive integers say $t+1, t+2, \dots, t+p$. In the same way G_2 is E-reverse super vertex magic then by Theorem 1.3, the weight of the vertex u , $\{w(u)/u \in V(G_2)\}$ consists of p integers say $s+1, s+2, \dots, s+p$.

Now here we adding these weights in the below manner

$$\begin{array}{cccccc}
 t+1, & t+2, & t+3, \dots, t+p-2, & t+p-1, & t+p \\
 s+p, & s+\frac{p-1}{2}, & s+p-1, \dots, s+\frac{p+3}{2}, & s+1, & s+\frac{p+1}{2}
 \end{array}$$

$$s+t+p+1, \quad s+t+\frac{p+3}{2}, \quad s+t+p+2, \dots, s+t+\frac{3p-1}{2}, \quad s+t+p, \quad s+t+\frac{3p+1}{2}$$

Thus $\{w(u)/u \in V(G)\} = \left\{ \frac{p+3}{2}, \frac{p+5}{2}, \dots, \frac{3p-1}{2}, \frac{3p+1}{2} \right\}$ consists of p consecutive integers. Hence by

Theorem 1.3, G is E-reverse super vertex magic.

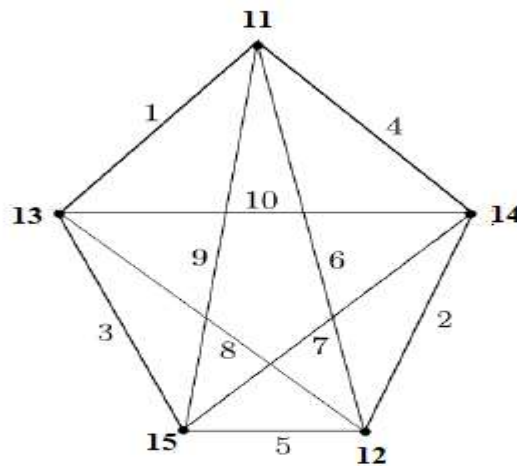


Fig. 8 E-super vertex magic labeling for $H_{4,5}(K_5)$, $p = 5$, $q = 10$ and $k = -9$

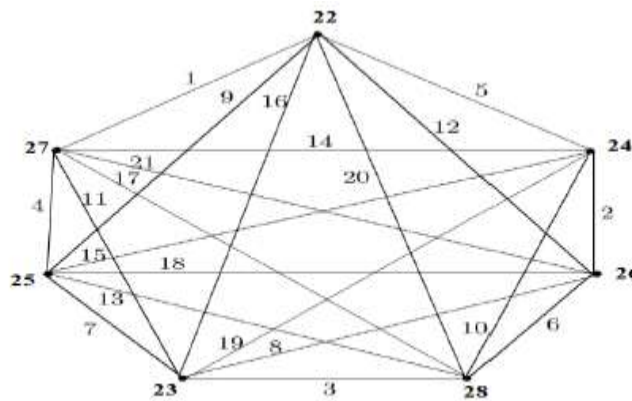


Fig. 9 E-super vertex magic labeling for $H_{6,7}$, $p = 7$, $q = 21$ and $k = -41$

In Fig 9, the graph $H_{6,7}$ can be decomposed into two E-reverse super vertex magic spanning subgraphs $H_{4,7}$ and C_7 . Therefore $H_{6,7}$ is E-reverse super vertex magic.

Theorem 3.8.8:- The graph $H_{m,n}$ is E-reverse super vertex magic if m is even and n is odd.

Proof:- We already observed that $H_{m,n}$ can be decomposed in to two spanning sub graphs $H_{m-2,n}$ and C_n . Again this sub graph $H_{m-2,n}$ can be decomposed into two spanning sub graphs $H_{m-4,n}$ and C_n . Again $H_{m-4,n}$ can be decomposed into two spanning sub graphs $H_{m-6,n}$ and C_n . Continuing in this process finally we get the two spanning sub graphs $H_{4,n}$ and C_n . We verified that $H_{4,n}$ and C_n are E-reverse super vertex magic if n is odd integer. Also by Theorem

3.8.7 , $H_{6,n}$ is E-reverse super vertex magic . Hence we have $H_{m,n}$ is E-reverse super vertex magic labelling if n is odd and m is even.

Corollary 3.8.9:- K_n is E-reverse super vertex magic if n is odd.

Proof:- Since $K_n = H_{n-1,n}$, so as an application of the Theorem 3.8 that K_n is E-reverse super vertex magic if n is odd.

Conjecture 3.8.10 :- If $n > 4$, $n \equiv 0 \pmod{4}$ and m is odd then $H_{m,n}$ has a E-reverse super vertex magic labelling.

From the Tables 1 and 2 , we uncertain that graph $H_{m,n}$ is E-reverse super vertex magic labelling if m is odd and $n \equiv 0 \pmod{4}$

Table 1 E-reverse super vertex magic labelling for $H_{7,8}$, $p=8$, $q=28$ and $k= - 296$

f(u)	29	31	35	33	32	30	36	34
29	0	15	13	28	12	25	3	2
31	15	0	5	4	14	18	17	27
35	13	5	0	22	26	7	21	10
33	28	4	22	0	9	8	20	11
32	12	14	26	9	0	16	1	23
30	25	18	7	8	16	0	19	6
36	3	17	21	20	1	19	0	24
34	2	27	10	11	23	6	24	0

Table 2 E-reverse super vertex magic labelling for $H_{11,12}$, $p=12$, $q=66$ and $k= - 69$

f(u)	70	78	68	71	74	75	73	67	72	77	76	69
70	0	31	33	48	4	47	44	57	23	9	53	17
78	31	0	11	61	64	40	8	56	1	41	29	32
68	33	11	0	37	46	15	21	16	60	36	30	59
71	48	61	37	0	35	25	3	18	64	24	39	12
74	4	64	46	35	0	5	43	45	66	10	14	38
75	47	40	15	25	5	0	34	52	20	54	28	51
73	44	8	21	3	43	34	0	58	49	19	63	27
67	57	56	16	18	45	52	58	0	7	26	6	22
72	23	1	60	65	66	20	49	7	0	62	13	2
77	9	41	36	24	10	54	19	26	62	0	42	50
76	53	29	30	39	14	28	63	6	13	42	0	55
69	17	32	59	12	38	51	27	22	2	50	55	0

IV.CONCLUSION

In this paper, we arrange some basic properties of E-reverse super vertex magic labelings and we establish E-reverse super vertex magic labelling of some families of graphs. We effort the focus of this paper is on the E-reverse super vertex magicness of $H_{m,n}$ and on some necessary conditions for a graph to be E-reverse super vertex magic.

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