ACCESSIBLE RINGS WITH WEAKLY NOVIKOV DENTITY

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ABSTRACT:In [5]Kleinfeldstudied that for an accessible ring every commutator is in the nucleus andassociator commutes with every element of *R*. Using these properties in this paper we prove that if *R* is a Prime Accessible ring with Weakly Novikov identity such that *T*⊆*N* where T_K=(((....((R,R),R)…,R),R),R) then the ring is associative or *T=0*. Next we prove that for Semiprime accessible ring with Weakly Novikov identity such that *T*⊆*N* and *(t,(R,R,R))*=0,then *R* is associative or *T*=0. Also we prove that if R is a prime accessible ring with Novikov identity such that [[R,R],R] is contained in nuclei, then R is associative or [[R,R],R]=0.

Key words: Accessible rings, Weakly Novikov Rings, Nucleus, Center.

1. INTRODUCTION

Throughout this paper we consider accessible rings with Weakly Novikov identity

$$(w,x,yz) = y(w,x,z) \quad (1)$$

For any ring R, let $T_K = (((\dots((R,R),R)\dots,R),R),R)$ where k is a +ve integer.

From the above we have $T_2 = (R,R) and\ T_3 = ((R,R),R)$ also $(R,T_K) = (T_K, R) \subseteq T_K$.

Obviously we have following identity

$$T_K + T_K R = T_K + R\,T_K \quad (2)$$

In any ring R, we have

$$(x,y,z) + (y,z,x) + (z,x,y) = (xy,z) + (yz,x) + (zx,y) \quad (3)$$

and Teichmuller identity

$$(wx,y,z) - (w,xy,z) + w,x,yz) = w(x,y,z) + (w,x,y)z \quad (4)$$

As a consequence of (4) we have$N$is associative sub ring of R.

Suppose that $w = n \in N$ in (4) we have

$$(nx,y,z) = n(x,y,z) \quad (5)$$

for all $x,y,z \in R$and $n \in N$.

Suppose that $z = m \in N$,in (4) we have

$$(w,x,ym) = (w,x,y)m \quad (6)$$

for all $w,x,y \in R$and$m \in N$.

Suppose that $x = j \in N$ in (4) we obtain

$$(wj, y, z) = (w, jy, z) \tag{7}$$

LEMMA 1: If R is an accessible ring with weakly Novikov identity, then $RN \subseteq N$ and

$$I.N = (R, R, R).N = 0.$$

PROOF: Let $z \in N$ and $w, x, y \in R$, Using (6) and (1) we have

$$(w, x, y)z = (w, x, yz)$$

$$= y\,(w, x, z)$$

$$= o$$

Thus $I.N = (R, R, R).N = 0$, and $RN \subseteq N$.♦

For any ring R, let $V_k = T_k + R\,T_k$ for all +ve integers k. In the sequel, for the convenience we denote T_k and V_k by T and V respectively.

LEMMA 2: If R is an accessible ring with weakly Novikov identity such that T is contained in the nuclei, then V is an ideal.

PROOF: From (2) we have $V = T + TR = T + RT$

Since T is contained in the nuclei, we have $TR \subset V$ and $RT \subset V$.

If T is in the nucleus, then $T + TR$ is in the ideal.

i.e. $VR = TR + TR^2 \subset V, RV = RT + R^2T \subset V.$

and $V + RV = V + VR$

Since of $RV = R(T + TR) = RT + RT.R \subset V + VR$.

And $VR = (T + RT)R$

$$= TR + R.TR \subset V + RV.$$

So V is an ideal.♦

2. MAIN RESULTS

THEOREM 1: If R is a prime accessible ring with weakly Novikov identity such that $T \subseteq N$, then R is associative or $T = 0$.

PROOF: Using $T \subseteq N$ and lemma (1) we get

$$I.V = I.(T + TR) = 0 \tag{8}$$

By lemma (2) and the primenessofR, (8) implies I=0or $V = 0$.

Thus, R is associative or $T = 0$.♦

LEMMA 3: If R is an accessible ring with weakly Novikov identity such that $T \subseteq N$ then $(R, R, T)R = 0$.

PROOF: We have $(R, T) = (T, R) \subseteq T$.

Using this, hypothesis, (4),(1),(7) and (5) for all $y \in T$, and $w, , x, z \in R$ we have

$$(w, x, y)z = w(x, y, z) + (w, x, y)z$$

$$= (wx, y, z) - (w, xy, z) + (w, x, yz)$$

$$= -(w, (x, y), z) - (w, yx, z) + y(w, x, z)$$

$$= -(wy, x, z) + y(w, x, z)$$

$$= -((w, y), x, z) - (yw, x, z) + y(w, x, z)$$

$$= 0.$$

hence we get $(R, R, T)R = 0$. (9)

This completes the proof of lemma.♦

THEOREM 2: Let R is a prime accessible ring with weakly Novikov identity such that $T \subseteq N$ and $(T, (R, R, R)) = 0$, then R is associative or $T = 0$.

PROOF: In accessible ring we have $J(x, y, z) \in N$ for all $x, y, z \in R$.

Using this, (3) and the hypothesis, for all $x \in T$ and $y, z \in R$ we get

$$(y, z, x) = (x, y, z) + (y, z, x) + (z, x, y)$$

$$= J(x, y, z) \in N.$$

Thus $(R, R, T) \subseteq N$.

Applying this, and (1),(9) we have

$$(R, R, RT)R = R(R, R, T).R$$

$$= R.(R, R, T)R$$

$$= 0.$$

Combining the above equation with (9) we have

$(R, R, V)R = 0$ (10)

Assume that $(T, (R, R, R)) = 0$.

Using this, (1),(9) and (4) and noting that $(T, R) \subseteq T$ for all w, x, y in $R$and z in T .

We have $(w, x, y)z.t = z(w, x, y).t$

$$= (w, x, zy)t$$

$$= (w, x, (z, y))\, t + (w, x, yz)\, t$$

$$= w(x, y, z).t + (w, x, y)z.t + (w, xy, z).t - (wx, y, z)t$$

$$= w(x, y, z).t + (w, x, y)z.t$$

And $(x, y, wz)t = w(x, y, z).t = 0$. (11)

Combining this with (9) we also obtain (10).

Using (1) and (10) we see that $< (R, R, T) > = (R, R, V)$.

By the semiprimeness of R, (10) implies $(R, R, V) = 0$.

By theorem (1), R is associative or $T = 0$.♦

THEOREM 3: If R is a prime accessible ring with weakly Novikov identity, then R is associative or commutative. In later case, $N = 0$or$R$ is associative.

PROOF: In view of theorem (1), we may assume that $(R, R) \subseteq N$.

Let$B = (B, R) + R(R, R)$.

By lemma (2) that B is an ideal of R.

Using lemma (3) we get $(R,R,(R,R))R = 0$.

Let $x \in (R,R)$, then we get $(y,z,x) \in N$.

Thus we obtain $(R,R,(R,R)) \subseteq N$.

Using this and (11) we obtain

$R(R,R,(R,R)).R = R.(R,R,(R,R))R = 0$.

Hence applying this, (1) and (11), and noting that B is an ideal of R, we obtain that

$(R,R,B).R=0$ *and* $<(R,R,(R,R))>=(R,R,B)$.

Thus, by semiprimeness of R we get $(R,R,B)=0$ and so $(R,R) \subseteq N$.

By theorem (1), R is associative or commutative.

Assume that R is commutative.

Thus we have $N R=RN \subseteq N$and $I. N= 0$ (By lemma (1)).

Hence N is an ideal of R.

By the primeness of R, $I. N = 0$ implies $I=0$ or $N= 0$.

i.e. $N = 0$ or R is associative.♦

In the sequel, for convenience we denote $v_3$by D.

LEMMA 4: If R is accessible ring with Weakly Novikov identity such that $[[R,R],R] \subseteq N$, then $<(R,R,D>.(R,R,R)=0$. where $<(R,R,D)> = (R, R, ,D)+(R,R,D)R+ R(R,R,D)$.

PROOF: Let $D = [[R,R],R]+R[[R,R],R]$ and $[[R,R],R] \subseteq N$.

By lemma (3), we obtain

$(R,R,[[R,R],R])R=0$ (12)

Thus (12) implies

$(R,R,[[R,R],R]) \subseteq N$ (13)

Assume that $y \in [[R,R],R]$ and $w,x,y,z,u,v,t \in R$.

Using (12), the hypothesis and (3) we have

$z(w,x,y) = [z,(w,x,y)]=[z, J(w,x,y)] \in [[R,R], R] \subseteq N$

and so by (1) twice we get

$(w,x,[z,y])+y(w,x,z)=(w,x,[z,y])+(w,x,yz)$

$=(w,x,zy)$

$=z(w,x,y)\in N$

Applying these, (1) and (13) we obtain

$(R,R, R[[R,R],R])=R (R,R, [[R,R],R]) \subseteq N$ (14)

$[[R,R],R]A = [[R,R],R](R,R,R)\subseteq N$ (15)

Then (15) implies

$[[R,R],R]A.R=[[R,R],R].AR$

$\subseteq [[R,R],R]A \subseteq N$ (16)

Combining (13) with (14) results in

$(R,R,D) \subseteq N$ (17)

Using (1), (15),(5) and (16),we have

$(w,x,yz)(u,v,t)=y(w,x,z).(u,v,t)$

$=(y(w,x,z).u,v,t)=0.$

Hence applying this, (2) and (12) we obtain

$(R,R,D)A=(R,R,D)(R,R,R)=0$ (18)

Then by (18), (17),(5) and (1) we get

$0= (R,R,D)(R,R,R)$

$= ((R,R,D)R,R,R)$

and $0=(R,R,D)(R,R,R)$

$=(R,R,(R,R,D)R)$

Thus by these, (12) and (1) we have

$R(R,R, [[R,R],R]).R=(R,R,D)R \subseteq N$ (19)

Let $x\in R(R,R, [[R,R],R])$ and $w,y,z\in R$.

Then by (14) and (1) we get $x\in N$ and $wx\in(R,R,D)$.

Hence by (7) and (17), we obtain $(w,xy,z)=(wx,y,z)=0$.

Combining this (1), (12) and (19) results in

$(R,R,D)R \subseteq N$ (20)

Using (1), (17) and (20) we see that

$<(R,R,D)>= (R,R,D)+ (R,R,D)R+ R.(R,R,D)R$

Combining (17) with (18) results in

$(R,R,D)R.A=(R,R,D).RA \subseteq(R,R,D)A=0$ (21)

applying (20) and (21), we get

$\{R.(R,R,D)R\}.A=R.\{(R,R,D)RA\} = 0.$

Thus using this, (18) and (21), we have $<(R,R,D)>.A = 0$.♦

THEOREM 4: If R is a prime accessible ring with Novikov identity such that $[[R,R],R]$ is contained in nuclei, then R is associative or $[[R,R],R]=0$.

PROOF: In view of theorem (1), we may assume that $[[R,R],R]\subseteq N$.

Let $D= [[R,R],R]+R[[R,R],R]$

By lemma (4) we obtain $<(R,R,D)>.A=0$,

where $<(R,R,D)>= (R,R,D)+(R,R,D)R+ R.(R,R,D)R$

By semiprime ness of R, this implies $<(R,R,D)>=0$.

Hence $[[R,R],R]\subseteq N$.

Then by theorem (1), R is associative or $[[R,R],R]=0$.♦

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