# A Study on Zagreb Indices And Co-Indices of Product of Cycle Graphs

U. Mary
*Associate Professor, Department of Mathematics,*
*Nirmala College for Women,*
*Coimbatore, Tamilnadu-641019, India*

K. Rajam
*Research Scholar,*
*Nirmala College for Women,*
*Coimbatore, Tamilnadu-641019, India.*

**S. Monolisa**
*Research Scholar,*
*Nirmala College for Women,*
*Coimbatore, Tamilnadu-641019, India.*

Abstract- **In this paper, we present a group of zagreb indices namely first and second zagreb indices, modified second zagreb index, first and second hyper zagreb indices, first, second and third redefined zagreb indices and augumented zagreb index of Lexicographical, Cartesian and Corona product of cycle graphs of order n and m. Moreover the co-indices of above said zagreb indices are also computed and we establish the relationship between them.**

**Keywords – Cartesian Product, Corona Product, Lexicographical Product, Zagreb co-indices, Zagreb indices.**

## I. Introduction

A topological index is defined as numerical parameter derived mathematically from the graph representing a molecule. The first and second Zagreb indices (introduced by Gutman and Trinajstic [8,9]) $M_1(G) = \sum_{uv\in E(G)}(d_u + d_v)$ & $M_2(G) = \sum_{uv\in E(G)}(d_u . d_v)$ were used to find total $\pi$ electron energy of molecules[9]. It receives a lot of importance in mathematical as well as chemical literature, because it has special characteristic that correlates the structural property and their molecular structure. Generalisation and extension of these indices were studied by G.H.Shirdel, H. Rezapour and A.M. Sayadi in [12,18]. Modified and hyper zagreb indices are defined as $M_2^*(G) = \sum_{uv\in E(G)}(\frac{1}{d_u . d_v})$, $HZ_1(G) = \sum_{uv\in E(G)}(d_u + d_v)^2$ & $HZ_2(G) = \sum_{uv\in E(G)}(d_u . d_v)^2$ Redefined version of Zagreb indices were introduced by Ranjini et.al[18], they are $ReZ_1(G)= \sum_{uv\in E(G)}(\frac{d_u + d_v}{d_u . d_v})$. $ReZ_2(G)= \sum_{e\in E(G)}(\frac{d_u . d_v}{d_u + d_v})$ $ReZ_3(G) = \sum_{uv\in E(G)}[(d_G(u) + d_G(v))(d_G(u) d_G(v))]$. Furtula et al. put forward the following modified version of the ABC index and named it as Augumented Zagreb index AZI (see [7]). It is defined as follows: $AZI(G) = \sum_{uv\in E(G)}\left[\frac{d_u . d_v}{d_u + d_v - 2}\right]^3$. It is also fascinating to pay attention to "**topological co-indices**" in which the sums involved run over the edges of the complement of G. The first Zagreb co-index is defined as (see [2,3]): $\overline{M_1(G)} = \sum_{uv \notin E(G)} (d_u + d_v)$ The second Zagreb coindex is defined as: $\overline{M_2(G)} = \sum_{uv \notin E(G)} (d_u . d_v)$ .Similarly we can have other Zagreb coindices. Graph operations play a vital role in the chemical graph theory and some chemically interesting graphs can be produced by different graph operations on graphs. Throughout this paper we considered three different products such as lexicographical, cartesian and corona of cycle graphs.

## II. LEXICOGRAPHICAL PRODUCT OF CYCLE OF GRAPHS

Degree of every vertex in $C_n \otimes C_m$ is 2+2m. Total number of vertices and edges in $C_n \otimes C_m$ are mn and nm(1+m) respectively. For finding Zagreb coindices we have to consider all non adjacent vertices in $C_n \otimes C_m$. If $C_n \otimes C_m$ is complete which has $\frac{nm(nm-1)}{2}$ edges. To compute the non-adjacent edges, let us subtract the adjacent edges of the given $C_n \otimes C_m$ from the assumed complete graph of $C_n \otimes C_m$.

i.e The no of non adjacent edges $= \frac{nm(nm-1)}{2} - nm(1+m)$

$= \frac{nm^2}{2}[(n-2)(m-3)]$.

THEOREM 2.1

The Zagreb indices and Zagreb co indices of $C_n \otimes C_m$ are,

(i) $M_1(C_n \otimes C_m)$ = 4nm (1+m)$^2$ , $\overline{M_1}(C_n \otimes C_m)$ = 2nm (1+m) [(n-2) (m-3)]

(ii) $M_2(C_n \otimes C_m)$ = 4nm (1+m)$^3$ , $\overline{M_2}(C_n \otimes C_m)$ = 2nm (1+m)$^2$ [(n-2)m - 3]

(iii) $M_2^*(C_n \otimes C_m) = \frac{nm}{4(1+m)}$ , $\overline{M_2^*}(C_n \otimes C_m) = \frac{nm}{8(1+m)^2}$[(n-2)m - 3]

(iv) $HZ_1(C_n \otimes C_m)$ = 16nm (1+m)$^3$ , $\overline{HZ_1}(C_n \otimes C_m)$ = 8nm (1+m)$^2$ [(n-2)m - 3]

(v) $HZ_2(C_n \otimes C_m)$ = 16nm (1+m)$^5$ , $\overline{HZ_2}(C_n \otimes C_m)$ = 8nm (1+m)$^4$ [(n-2)m - 3]

(vi) $ReZ_1(C_n \otimes C_m)$ =nm , $\overline{ReZ_1}(C_n \otimes C_m) = \frac{nm}{2}\left[\frac{[(n-2)\,m-3]}{(1+m)}\right]$

(vii) $ReZ_2(C_n \otimes C_m)$ =nm (1+m)$^2$ , $\overline{ReZ_2}(C_n \otimes C_m) = \frac{nm}{2}[(n-2)m-3](1+m)$

(viii) $ReZ_3(C_n \otimes C_m)$ = 16 nm (1+m)$^4$, $\overline{ReZ_3}(C_n \otimes C_m)$ = 8nm (1+m)$^3$ [(n-2) m-3]

(ix) $AZI(C_n \otimes C_m)$ = nm (1+m)$^4$ , $\overline{AZ_1}(C_n \otimes C_m) = \frac{nm}{2}[(n-2)m-3](1+m)^3$

Relation between the Zagreb indices and Zagreb co-indices of $C_n \otimes C_m$ are established as follows:

(i) $M_1(C_n \otimes C_m) + \overline{M_1}(C_n \otimes C_m)$ = 2nm (1+m) (nm-1)

(ii) $M_2(C_n \otimes C_m) + \overline{M_2}(C_n \otimes C_m)$ = 2nm (1+m)$^2$ (nm-1)

(iii) $HZ_1(C_n \otimes C_m) + \overline{HZ_1}(C_n \otimes C_m)$ = 8nm (1+m)$^2$ (nm-1)

(iv) $HZ_2(C_n \otimes C_m) + \overline{HZ_2}(C_n \otimes C_m)$ = 8nm (1+m)$^4$ = $\frac{(nm-1)}{2}$ (nm-1)

(v) $ReZ_1(C_n \otimes C_m) + \overline{ReZ_1}(C_n \otimes C_m) = \frac{nm(nm-1)}{2(1+m)}$

(vi) $ReZ_2(C_n \otimes C_m) + \overline{ReZ_2}(C_n \otimes C_m)$ = nm (1+m) $\frac{(nm-1)}{2}$

(vii) $ReZ_3(C_n \otimes C_m) + \overline{ReZ_3}(C_n \otimes C_m)$ = 16 nm (1+m)$^3 \frac{(nm-1)}{2}$

(viii) $AZ_1(C_n \otimes C_m) + \overline{AZ_1}(C_n \otimes C_m)$ = nm (1+m)$^3 \frac{(nm-1)}{2}$

Results:

(i) $M_2(C_n \otimes C_m) + \overline{M_2}(C_n \otimes C_m) = (1+m)[M_1(C_n \otimes C_m) + \overline{M_1}(C_n \otimes C_m)]$

(ii) $\overline{M_2^*}(C_n \otimes C_m) = \frac{M_2^*(C_n \otimes C_m)}{2(1+m)}[(n-2)(m-3)]$

(iii) $HZ_2(C_n \otimes C_m) + \overline{HZ_2}(C_n \otimes C_m) = (1+m)^2[HZ_1(C_n \otimes C_m) + \overline{HZ_1}(C_n \otimes C_m)]$

(iv) $ReZ_2(C_n \otimes C_m) + \overline{ReZ_2}(C_n \otimes C_m) = (1+m)^2[ReZ_1(C_n \otimes C_m) + \overline{ReZ_1}(C_n \otimes C_m)]$

(v) $ReZ_3(C_n \otimes C_m) + \overline{ReZ_3}(C_n \otimes C_m) = 16[AZ_1(C_n \otimes C_m) + AZ_1(C_n \otimes C_m)]$

## III. Cartesian Product of Cycle of Graphs

Total number of vertices and edges in $C_n \square C_m$ are calculated as nm and 2mn respectively. Every edge set has the degree set(4,4) Finding the number of non-adjacent edges in the cycle graphs $C_n \square C_m$, consider complete graph of $C_n \square C_m$ which has $\frac{nm(nm-1)}{2}$ edges. The no of non adjacent edges in $C_n \square C_m$ is $\left[\frac{nm(nm-1)}{2} - 2mn\right]$ that is $\frac{nm(nm-5)}{2}$.

Theorem 3.1

The zagreb indices and co-indices of $C_n \square C_m$ are

(i) $M_1(C_n \square C_m) = 16nm$ , $\overline{M_1}(C_n \square C_m) = 4nm(nm-5)$

(ii) $M_2(C_n \square C_m) = 32nm$ , $\overline{M_2}(C_n \square C_m) = 8nm(nm-5)$

(iii) $M_2^*(C_n \square C_m) = \frac{nm}{8}$ , $\overline{M_2^*}(C_n \square C_m) = \frac{nm(nm-5)}{32}$

(iv) $HZ_1(C_n \square C_m) = 128nm$ , $\overline{HZ_1}(C_n \square C_m) = 32nm(nm-5)$

(v) $HZ_2(C_n \square C_m) = 512nm$ , $\overline{HZ_2}(C_n \square C_m) = 128nm(nm-5)$

(vi) $ReZ_1(C_n \square C_m) = nm$ , $\overline{ReZ_1}(C_n \square C_m) = \frac{nm(nm-5)}{4}$

(vii) $ReZ_2(C_n \square C_m) = 4nm$ , $\overline{ReZ_2}(C_n \square C_m) = 4nm(nm-5)$

(viii) $ReZ_3(C_n \square C_m) = 256nm$ , $\overline{ReZ_3}(C_n \square C_m) = 64nm(nm-5)$

(ix) $AZ_1(C_n \square C_m) = 16nm$ , $\overline{AZ_1}(C_n \square C_m) = 4nm(nm-5)$

Relation between Zagreb indices and Zagreb coindices are established in the following equations

(i) $M_1(C_n \square C_m) + \overline{M_1}(C_n \square C_m) = 4nm(nm-1)$

(ii) $M_2(C_n \square C_m) + \overline{M_2}(C_n \square C_m) = 8nm(nm-1)$

(iii) $HZ_1(C_n \square C_m) + \overline{HZ_1}(C_n \square C_m) = 32nm(nm-1)$

(iv) $HZ_2(C_n \square C_m) + \overline{HZ_2}(C_n \square C_m) = 128nm(nm-1)$

(v) $ReZ_1(C_n \square C_m) + \overline{ReZ_1}(C_n \square C_m) = \frac{nm(nm-1)}{4}$

(vi) $ReZ_2(C_n \square C_m) + \overline{ReZ_2}(C_n \square C_m) = nm(nm-1)$

(vii) $ReZ_3(C_n \square C_m) + \overline{ReZ_3}(C_n \square C_m) = 64nm(nm-1)$

(viii) $AZ_1(C_n \square C_m) = 16nm$ and $\overline{AZ_1}(C_n \square C_m) = 4nm(nm-1)$

Results:

(i) $M_2(C_n \square C_m) + \overline{M_2}(C_n \square C_m) = 2[M_1(C_n \square C_m) + \overline{M_1}(C_n \square C_m)]$

(ii) $\overline{M_2^*}(C_n \square C_m) = M_2^*(C_n \square C_m)\frac{(nm-5)}{4}$

(iii) $HZ_2(C_n \square C_m) + \overline{HZ_2}(C_n \square C_m) = 4[HZ_1(G) + \overline{HZ_1}(G)]$

(iv) $ReZ_2(C_n \square C_m) + \overline{ReZ_2}(C_n \square C_m) = 4[ReZ_1(G) + \overline{ReZ_1}(G)]$

(v) $ReZ_3(C_n \square C_m) + \overline{ReZ_3}(C_n \square C_m) = 32[AZ_1(G) + AZ_1(G)]$

## IV. CORONA PRODUCT OF CYCLE OF GRAPHS

Theorem 4.1

Case:1

Cycle graph of order n and m, if n≠m;

The number of vertices and edges in $(C_n \odot C_m)$ are calculated as n+nm and n+2nm respectively. The edge subdivisions in $(C_n \odot C_m)$ are $|E_{2+m,2+m}| = n$, $|E_{3,2+m}| = nm$, $|E_{3,3}| = nm$

The Zagreb indices and Zagreb co-indices of $(C_n \odot C_m)$ are,

(i) $M_1(C_n \odot C_m) = n[4+13m+m^2]$, $\overline{M_1}(C_n \odot C_m) = n(n-3)(2+m)+mn(5n-m-8+2nm)$

(ii) $M_2(C_n \odot C_m) = n(2+m)^2 + 3mn[5+m]$,

$\overline{M_2}(C_n \odot C_m) = .\frac{n^2-3n}{2}(m+2)^2 + nm(n-1)3(m+2) + 9nm\,\frac{mn-3}{2}$

(iii) $M_2^*(C_n \odot C_m) = \frac{9n+10nm+7nm^2+nm^3}{9(2+m)^2}$

$\overline{M_2^*}(C_n \odot C_m) = \frac{n(n-3)}{2(m+2)^2} + \frac{nm(n-1)}{3(m+2)} + \frac{mn(mn-3)}{18}$

(iv) $HZ_1(C_n \odot C_m) = n(4+2m)^2 + mn(5+m)^2 + 36mn$

$\overline{HZ_1}(C_n \odot C_m) = \frac{n^2-3n}{2}(2+m+4)^2 + nm(n-1)(5+m)^2 + 18mn\left(\frac{nm-3}{2}\right)$

(v) $HZ_2(C_n \odot C_m) = n(2+m)^2 + 3mn(2+m) + 9mn$

$\overline{HZ_2}(C_n \odot C_m) = \frac{n(n-3)}{2}(2+m)^2 + nm(n-1)3(m+2) + 9mn\left(\frac{nm-3}{2}\right)$

(vi) $ReZ_1(C_n \odot C_m) = \frac{2n+mn(m+3)}{2+m}$

$\overline{ReZ_1}(C_n \odot C_m) = \frac{n^2-3n}{2+m} + mn\left[\frac{(n-1)(5+m)}{3m+6} + \frac{mn-3}{3}\right]$

(vii) $ReZ_2(C_n \odot C_m) = \frac{n}{2}(m+2) + 3mn\left[\frac{9+3m}{2(5+3m)}\right]$

$\overline{ReZ_2}(C_n \odot C_m) = \frac{n^2-3n}{2}\left(\frac{2+m}{2}\right) + \frac{nm(n-1)3(m+2)}{3m+5} + \frac{mn(mn-3)}{2}\left(\frac{3}{2}\right)$

(viii) $ReZ_3(C_n \odot C_m) = 2n(2+m)^3 + 3mn(m^2 + 7m + 28)$

$\overline{ReZ_3}(C_n \odot C_m) = (n^2-3n)(2+m)^{3+}3mn(10n + m^2n + 16mn - m^2 - 7m - 37)$

(ix) $AZ_1(C_n \odot C_m) = n\left(\frac{2+m}{2}\right)^3 + 27mn\left(\frac{2+m}{5+m}\right)^3 + nm\left(\frac{27}{8}\right)$

$\overline{AZ_1}(C_n \odot C_m) = \frac{n^2-3n}{2}\left(\frac{2+m}{2}\right)^3 + 27mn(n-1)\left(\frac{2+m}{5+m}\right)^3 + \frac{mn}{2}(mn-3)\frac{27}{8}$

Relation between Zagreb indices and Zagreb co-indices are established in the following equations

(i) $M_1(C_n \odot C_m) + \overline{M_1}(C_n \odot C_m) = (2+m)n(n-1) + (5+m)n^2m + 3mn(nm-1)$

(ii) $M_2(C_n \odot C_m) + \overline{M_2}(C_n \odot C_m) = (m+2)^2\left(\frac{n(n-1)}{2}\right) + 3(2+m)(n^2m) + = \frac{9}{2}(nm-1)mn$

(iii) $M_2^*(C_n \odot C_m) + \overline{M_2^*}(C_n \odot C_m) = \frac{2nm^3+8n^2m^2+2mn+9n^2-9n+8nm^2+12n^2m}{18(m+2)^2}$

(iv) $HZ_1(C_n \odot C_m) + \overline{HZ_1}(C_n \odot C_m) = (4+2m)^2\left(\frac{n(n-1)}{2}\right) + (5+m)^2 n^2m + 36nm\left(\frac{(nm-1)}{2}\right)$

(v) $HZ_2(C_n \odot C_m) + \overline{HZ_2}(C_n \odot C_m) = (2+m)^2\left(\frac{n(n-1)}{2}\right) + 3mn(m+2) + 9mn\left(\frac{(nm-1)}{2}\right)$

(vi) $ReZ_1(C_n \odot C_m) + \overline{ReZ_1}(C_n \odot C_m) = \frac{n(n-1)}{2+m} + n^2 m\left(\frac{(5+m)}{6+3m}\right) + \frac{nm(nm-1)}{3}$

(vii) $ReZ_2(C_n \odot C_m) + \overline{ReZ_2}(C_n \odot C_m) = \frac{2+m}{2}\left(\frac{n^2-n}{2}\right) + \frac{3n^2 m(m+2)}{5+3m} + \frac{3}{2}\left(\frac{mn-1}{2}\right) mn$

(vii) $ReZ_3(C_n \odot C_m) + \overline{ReZ_3}(C_n \odot C_m) = (2+m)^3\left[(n^2-n) + 3mn\,(m^2 n + 16mn + 10n\_9)\right]$

(viii) $AZ_1(C_n \odot C_m) + \overline{AZ_1}(C_n \odot C_m) = \left(\frac{2+m}{2}\right)^3 n\left(\frac{(n-1)}{2}\right) + 27n^2 m\left(\frac{2+m}{2}\right)^3 + \frac{27}{8} m\left(\frac{mn-1}{2}\right)$

Case: 2

Cycle graph of order n and m, if n=m;

The number of vertices and edges in $(C_n \odot C_m)$ are calculated as $n+n^2$ and $n+2n^2$ respectively. The edge subdivisions in $(C_n \odot C_m)$ are $|E_{2+n,2+n}| = n$, $|E_{3,2+n}| = n^2$, $|E_{3,3}| = n^2$

(i) $M_1(C_n \odot C_m) = n^3 + 3n^2 + 4n$ and $\overline{M_1}(C_n \odot C_m) = 4n^4 + 5n^3 - 15n^2 - 6n$

(ii) $M_2(C_n \odot C_m) = n(4n^2 + 19n + 4)$ and $\overline{M_2}(C_n \odot C_m) = \frac{n}{2}[16n^3 + 17n^2 - 47n - 12]$

(iii) $M_2^*(C_n \odot C_m) = \frac{n}{(2+n)^2}[n^3 + 7n^2 + 10n + 9]$

$\overline{M_2^*}(C_n \odot C_m) = \frac{1}{18(2+n)^2}[n^6 + 4n^5 + 7n^4 - 6n^3 - 15n^2 - 27n]$

(iv) $HZ_1(C_n \odot C_m) = n^4 + 14n^3 + 77n^2 + 16n$

$\overline{HZ_1}(C_n \odot C_m) = [n^5 + 27n^4 + 17n^3 - 95n^2 - 24n]$

(v) $HZ_2(C_n \odot C_m) = n^5 + 17n^4 + 60n^3 - 149n^2 - 16n$

$\overline{HZ_2}(C_n \odot C_m) = \frac{1}{2}[23n^5 + 135n^4 - 40n^3 - 395n^2]$

(vi) $ReZ_1(C_n \odot C_m) = \frac{n}{2+n}[n^3 + 3n^2 + 2]$

$\overline{ReZ_1}(C_n \odot C_m) = \frac{n}{3(2+n)}[n^4 + 3n^3 - 2n^2 - 5n - 9]$

(vii) $ReZ_2(C_n \odot C_m) = \frac{n}{2(5+n)}[10n^2 + 34n + 10]$

$\overline{ReZ_2}(C_n \odot C_m) = \frac{n}{4(5+n)}[4n^4 + 31n^3 - 8n^2 - 99n]$

(viii) $ReZ_3(C_n \odot C_m) = 5n^4 + 33n^3 + 108n^2 + 16n$

$\overline{ReZ_3}(C_n \odot C_m) = 4n^5 + 48n^4 + 3n^3 - 139n^2 - 24n$

(ix) $AZI(C_n \odot C_m) = \frac{n}{8(5+4)^3}[n^6 + 21n^5 + 420n^4 + 2464n^3 + 6387n^2 + 7303n1000]$

$\overline{AZI}(C_n \odot C_m) = \frac{n}{16(5+4)^3}[81n^6 + 1069n^5 + 3585n^4 + 1542n^3 - 13103n^2 - 16581n + n^8 + 18n^7]$

Relation between Zagreb indices and Zagreb co-indices are established in the following equations

(i) $M_1(C_n \odot C_m) + \overline{M_1}(C_n \odot C_m) = 4n^4 + 6n^3 - 2n^2 - 2n$

(ii) $M_2(C_n \odot C_m) + \overline{M_2}(C_n \odot C_m) = \frac{1}{2}[16n^4 + 15n^3 - 9n^2 - 4n]$

(iii) $M_2^*(C_n \odot C_m) + \overline{M_2^*}(C_n \odot C_m) = \frac{n}{18(2+n)^2}[9n^3 + 8n^2 + 5n - 9 + 4n^4 + n^5]$

(iv) $HZ_1(C_n \odot C_m) + \overline{HZ_1}(C_n \odot C_m) = n^5 + 28n^4 + 32n^3 - 18n^2 - 8n$

(v) $HZ_2(C_n \odot C_m) + \overline{HZ_2}(C_n \odot C_m) = \frac{1}{2}[-246n^4 + n^6 + 154n^4 + 20n^3 + 24n^5 - 32n]$

(vi) $ReZ_1(C_n \odot C_m) + \overline{ReZ_1}(C_n \odot C_m) = \frac{n}{3(2+n)}[n^4 + 6n^3 + 7n^2 - 5n - 3]$

(vii) $ReZ_2(C_n \odot C_m) + \overline{ReZ_2}(C_n \odot C_m) = \frac{n}{4(5+n)}[4n^4 + 12n^2 + 31n^3 - 31n + 2]$

(viii) $ReZ_3(C_n \odot C_m) + \overline{ReZ_3}(C_n \odot C_m) = 4n^5 + 53n^4 + 36n^3 - 31n^2 - 8n$

(ix) $AZ_1(C_n \odot C_m) + \overline{AZ_1}(C_n \odot C_m) = \frac{n}{16(5+n)^3}\left[\begin{matrix} n^8 + 18n^7 + 83n^6 + 1111n^5 + 4425n^4 + \\ 6470n^3 - 329n^2 - 2175n \end{matrix}\right]$

## IV.CONCLUSION

In this paper, we have found Zagreb group of (degree based) topological indices and co-indices for three different product of cycle graphs. Some remarkable results are also established. These results also can be extended for tensor product of cycle graphs.

REFERENCES

[1] Anitha .J, Maheswari .B, “Total edge dominating functions of corona product graph of a Cycle with a complete graph”, 12(2019), 12-16.

[2] A. R. Ashrafi, T. Doˇsli´c, A. Hamzeh,” The Zagreb coindices of graph operations”, *Discr. Appl. Math*. 158 (2010) 1571–1578.

[3] A. R. Ashrafi, T. Doˇsli´c, A. Hamzeh, Extremal graphs with respect to the Zagreb coindices, MATCH Commun. Math. Comput. Chem. 65 (2011) 85–92.

[4] De .N, Nayeem S. M. A, and Pal .A , “F-index of some graph operations”, *Discrete Math. Alg. Appl.,* (2016).

[5] Furtula B, Gutman I, “Forgotten topological index”., *J. Math.* Chem., 53, (2015).

[6] Furtula B, Das K. C, and Gutman I, “Comparative analysis of symmetric division deg index as potentially useful molecular descriptor”, *Int J Quantum Chem.,* (2018),

[7] B. Furtula, A. Graovac, D. Vukicevic, “Augmented Zagreb index”, *J. Math. Chem*., 48 (2010), 370–380.

[8] GutmanI, TrinajstićN, “Graph theory and molecular or bitals. Total π-electronenergy of alter-nant hydrocarbons”, *Chem. Phys. Lett.* 17(4) (1972) 535–538.

[9] Gutman I, “Degree-based topological indices” *Croat. Chem. Acta,* 86(4)(2013), 351–361.

[10] I. Gutman and K. C. Das, “The first Zagerb index 30 years after”, *MATCH Commun. Math. Comput. Chem.*, vol 50, 83-92, (2004).

[11] Harary F, “Graph theory”, *Narosa Publishing House,* New Delhi, 1969.

[12] G.H. Shirdel, H. Rezapour, A.M. Sayadi,” The hyper-Zagreb index of graph operations”, *Iran. J. Math. Chem*., 4, No 2 (2013), 213–220.

[13] Hosamani S, Perigidad D, Maled S.J.Y, and Gavade S, “QSPR Analysisof Certain Degree Based Topological Indices”, *J. Stat. Appl. Pro.,* 6,(2), (2017), 1-11.

[14] Randic M, “On characterization of Molecular branching”., *J. Am. Chem. Soc* (1975) 97(23)., 6609—6615.

[15] Rajam K, Monolisa S and Mary U,“Topological descriptors of product of complete graphs”, *AIP conference proceedings.,2261,*(2020) 030056(1-6).

[16] Trinajstic N, “Chemical graph theory”, *CRC Press, Bo Ca Raton, FL* (1992).

[17] Vukicevic .D and Gasperov.M, “Bondaddtitive modelying 1. Adriatic indices”, *Croat.Chem.Acta* 83(2010), 243-260.

[18] Vukicevic .D “Bondaddtitive modelying 2.mathematical propertices of max-minrodeg index”, *Croat.Chem.Acta 54*(2010), 261-273.

[19] Wei Gao, Weifan wang and Farahani M.R, “Topological indices study of molecular structure in Anticancer drugs”, *Journal of chemistry.,*(2016).

[20] Xiujun Zhang, Huiqin Jiang, Jia-Bao-Liu, and Zehui Shao, “The Cartesian product and Join graphs on edge-version atom-bond connectivity and geometric arithmetic indices”, *molecules*., 23,(2018), 1-15.

[21] Zhou,N.Trinajstic., “On novel connectivity index”., *J.Math,chem*., 46, 2009,1252-1270.