Performance Modeling & Reliability Analysis of Demand and Supply Model

Shalini Jindal, Dr. Reena Garg, Dr. Tarun Kumar Garg, Yashasvi Garg

Abstract-In the present paper, we have computed the reliability of the Demand and Supply Model with season-wise variation victimization CAS Mathematica & also enhance the performance of the system. Modeling of the system is based on Markov Method and set of differential equations are formed by using Chapman-Kolmogorov differential equation. The model consists of three operating states, one down state and one failed state. According to demand of season, system is in their respective state. when demand is reduced to negligible then produced items exit in stock which is utilized further on demand of item. The performance of every unit of the system has been analysed for better output. The optimum reliability achieved is nearly 90% with best fitting of failure and repair rates. Graphs are plotted to show the variation of reliability.

Keywords- Chapman Kolmogorov Differential equations, Demand and Supply Model, Mathematica, Reliability, Semi Markov Process.

1 Introduction:

THE present paper analyses the variation of demand during season either winter or summer. During winter season demand of tea, coffee, soup etc are increased and demand of Coldrinks, Thandie, Sharbat etc. are decreased slowly but when winter season at peak then their demand is negligible. In that case produced items exit in stock due to off-season which are further utilized in on-season. Similarly, during summer season demand of Coldrinks, Thandie, Sharbat etc are increased and demand of tea, coffee, soup etc. are decreased slowly but when summer season at peak then their demand is negligible. In that case produced items exit in stock due to off-season which are further utilized in on-season. For computation of reliability, its needs modelling of the system under real working condition to measure its performance.

In [1] Reena Garg has analysed the "M.T.T.F. and Reliability vs Time for a Milk Manufacturing and Processing Unit" using Boolean Function Technique. In [2] Tarun Garg has analysed the "Application of CAS Maxima to the reliability of Skimmed Milk Powder Production System of a Dairy Plant" using Mathematica. In [3] Reena Garg has analysed the "Reliability Evaluation for Power Distribution System". In [4] Kumar, J. Kadyan, M. S., Malik, S. C. & Jindal, C. have analysed "Reliability Measures of a Single-Unit System Under Preventive Maintenance and Degradation with Arbitrary Distributions of Random Variables".In [6,7,10] Khanduja has analysed steady state behaviour of paper plant, crystallization of sugar plant using G.A. and availability of paper plant. The paper includes six sections. The present section is introductory type includes the literature review. Second section presents the ''Mathematical aspects of reliability''. The third section involved with ''System description, notations and assumptions''. Fourth section deals with ''Mathematical formulation of demand and supply system of a dairy plant''. Fifth section concerns with ''Performance analysis of the system''. Finally, last section deals with ''Discussion and conclusion''.

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2 Mathematical Aspects of Reliability

2.1 Reliability

Reliability of a unit (or product) is the probability that a failure may not occur in a given time interval. Reliability stresses four elements namely:

1.Probability
2.Intended function
3.Time
4.Operating conditions

The reliability of a component may be calculated as:

$$R(t) = 1- e^{-\alpha t}$$

where α is the constant failure rate of the component (per hour) and t is the operation time (hour).

2.2 Markov approach

Khanduja (2008,2012), Singh and Kumar (2010a) and Kumar, Kadayan and Malik (2014) used the Markov approach for availability analysis of different process plants. According to

Markov, rate of probability of staying in any state is equal to the sum of frequencies of entry into that state minus the sum of frequencies departure from that state if P0(t) represents the probability of zero occurrences in time t, the probability of zero occurrences in time (t + △t) is given by the Eq. (1)

$P0\ (t + \triangle t) = (1 - \alpha \triangle t)\ P0(t)$ (1)

Similarly,

$P1\ (t + \triangle t) = \beta \triangle t\ P0(t) + (1 - \alpha \triangle t)\ P1(t)$ (2)

Where α is the failure rate and β is the repair rate of the component or subsystem respectively.

After simplifying and taking △t → 0, the Eq. (2) is reduced to

$P1'(t) + \beta P1(t) = \alpha Po(t)$ (3)

Using the concept employed in Eq. (3), the equations for transient states are derived.

3 System Configuration, Notations and Assumptions

The demand and supply model have following units:

(1) Unit S0 (Operating unit): In this unit, winter and summer products both are produced.

(2) Unit S1 (Operating unit of winter season): In this unit, demand of winter products is greater than or equal to production. when it is not in operated state then it goes to failed state S4.

(3) Unit S2 (Operating unit of summer season*)*: In this unit, demand of summer products is greater than or equal to production. when it is not in operated state then it goes to failed state S4.

(4) Unit S3 (Down unit): In this unit, off season products exist in stock, when there is no demand.

(5) Unit S4 (Failed unit): In this unit, operated state is in failed state.

Notations:

- Operative state of the system
- Down state of the system
- Failed state of the system

- S_0 = Unit is in operative state where winter and summer items produced.
- S_1 = Unit is in operative state when demand of winter products is greater than or equal to production.
- S_2 = Unit is in operative state when demand of summer products is greater than or equal to production.
- S_3 = Unit is in down state having no demand.
- S_4 = Unit is in failed state.
- a_1= Rate of increase of demand so as to become greater than or equal to production.
- a_2 = Rate of decrease of demand so as to become less than production.
- a_3 = Rate of going from upstate to downstate, when demand is less than production & production goes on increasing and as a result, we have lot of production in the stock. This production needs to be stopped.
- a_4 = Rate of change of state from down to up when there is no production with the system but demand is there.
- b = Constant failure rates of S_1 and S_2.
- w = Constant repair rates of S1 and S2 from failed to full working state.
- Pj(t) (j=1, 2, 3, ..., 13) = Represents the probability that the system is in jth state at time t.
- Pj ′(t) (j=1, 2, 3, ..., 13) = Represents derivative with respect to t.

Assumptions:

- System is analysed with season wise variation for transient state.
- Individual repairmen are available for repair of sever.
- Repaired unit is as good as new one.
- Repair and failure rates are independent of each other.
- Failure and repaired rates are exponentially distributed.

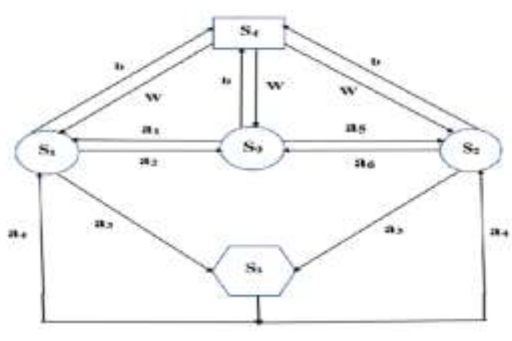


Fig. 1 : Numerical analysis of reliability of Demand and Supply Model

4 Mathematical formulation of the system

The mathematical modelling of the system is carried out to determine the reliability of the demand and supply model and following chapman Kolmogorov differential equations are developed on basis of Markov birth-death process are:

$$P_0'[t] = -H_1 * P_0[t] + a_2 * P_1[t] + a_6 * P_2[t] + w * P_4[t] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$
$$P_1'[t] = -H_2 * P_1[t] + a_1 * P_0[t] + w * P_4[t] + a_4 * P_3[t] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$
$$P_2'[t] = -H_3 * P_2[t] + a_5 * P_0[t] + w * P_4[t] + a_4 * P_3[t] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$
$$P_3'[t] = -H_4 * P_3[t] + a_3 * P_1[t] + a_3 * P_2[t] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$
$$P_4'[t] = -H_5 * P_4[t] + b * P_0[t] + b * P_1[t] + b * P_2[t] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Where $H_1 = (a_1 + a_5 + b)$

$H_2 = (a_2 + a_3 + b)$

$H_3 = (a_3 + a_6 + b)$

$H_4 = (2 * a_4)$

$H_5 = (3 * w)$

With initial conditions at time $t = 0$

$$P_i(t) = \begin{cases} 1 \text{ for } i = 0 \\ 0 \text{ for } i \neq 0 \end{cases} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

The system of differential equations with initial conditions are solved by using CAS Mathematica

Values of P_0, P_1, P_2, P_3, P_4 given by Mathematica at point 't':

$\mathbf{P_0[t]} = 0.064\, e^{(-3.43)t} (0.180\, e^{(1.41)t} + 14.123\, e^{(2.35)t} + 6.48 * 10^{-33}\, e^{(3.12)t} + 0.285\, e^{(3.41)t} + e^{(3.43)t})$,

$\mathbf{P_1[t]} = 0.316\, e^{(-3.43)t} (-0.185\, e^{(1.41)t} - 1.12\, e^{(2.35)t} - 1.77 * 10^{-17}\, e^{(3.12)t} + 0.303\, e^{(3.41)t} + e^{(3.43)t})$,

$\mathbf{P_2[t]} = 0.316\, e^{(-3.43)t} (-0.185\, e^{(1.41)t} - 1.12\, e^{(2.35)t} - 1.77 * 10^{-17}\, e^{(3.12)t} + 0.303\, e^{(3.41)t} + e^{(3.43)t})$,

$\mathbf{P_3[t]} = 0.070\, e^{(-3.43)t} (1.50\, e^{(1.41)t} - 2.81\, e^{(2.35)t} + 1.39 * 10^{-32}\, e^{(3.12)t} - 0.31\, e^{(3.41)t} + e^{(3.43)t})$,

$\mathbf{P_4[t]} = 0.232\, e^{(-3.43)t} (0.001\, e^{(1.41)t} - 0.004\, e^{(2.35)t} - 1.20 * 10^{-32}\, e^{(3.12)t} - 0.997\, e^{(3.41)t} + e^{(3.43)t})$

The reliability of the system can be computed by

$R(t) = P_0(t) + P_1(t) + P_2(t) + P_3(t)$ ……………(10)

5. Performance analysis of the system: The reliability of system is calculated by using equation (7) for various values of failure and
repair rates for transient state and given in tables as

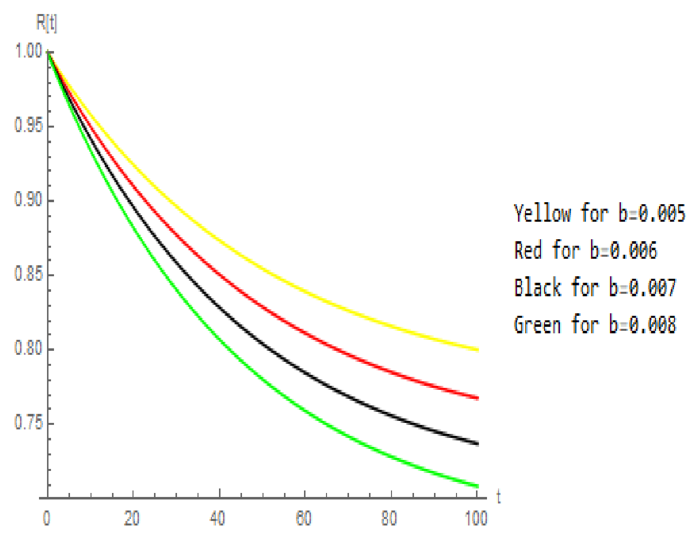
Case I:
When $a_1 = a_5$, $a_2 = a_6$; demand of summer and winter products are approximately same.

Effect of failure rate on reliability of the system

The reliability of the system is studied by varying their values as; b = 0.005, 0.006, 0.007, 0.008 and w =0.005, $a_1 = a_5$ =0.5, $a_2 = a_6$ = 0.1, a_3 = 0.2, a_4 =0.9.

Table 1: Effect of Failure rate on reliability of the system.

R(t) T	b = .005		b = .007	b = .008
10	0.95821	b = .006	0.942017	0.934029
20	0.924384	0.910034	0.895932	0.882074
30	0.896559	0.877395	0.858707	0.840485
40	0.873673	0.850789	0.828638	0.807194
50	0.854847	0.829103	0.804349	0.780545
60	0.839363	0.811426	0.784729	0.759213
70	0.826626	0.797017	0.768881	0.742138
80	0.816149	0.785271	0.75608	0.728469
90	0.807531	0.775698	0.745739	0.717528
100	0.800443	0.767894	0.737386	0.70877



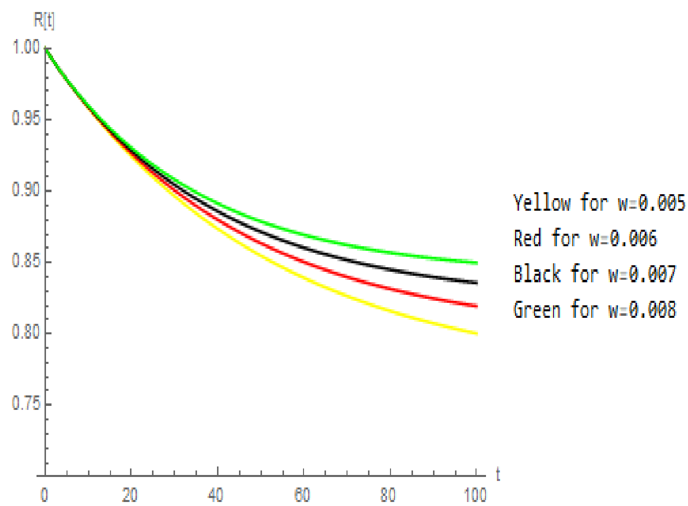
Graph - 1

Effect of repair rate on reliability of the system:

The reliability of the system is studied by varying their values as; w = 0.005, 0.006, 0.007, 0.008 and b =0.005, $a_1 = a_5$ =0.5, $a_2 = a_6$ = 0.1, a_3 = 0.2, a_4 =0.9.

Table2:Effect of repair rate on reliability of the system.

R(t) \ T	w = .005	w = .006	w = .007	w= .008
10	0.95821	0.958817	0.959412	0.959996
20	0.924384	0.926476	0.928489	0.930426
30	0.896559	0.90066	0.904534	0.908197
40	0.873673	0.880052	0.885977	0.891485
50	0.854847	0.863602	0.871602	0.878922
60	0.839363	0.850471	0.860466	0.869477
70	0.826626	0.839989	0.851839	0.862377
80	0.816149	0.831622	0.845156	0.857039
90	0.807531	0.824943	0.839979	0.853026
100	0.800443	0.819611	0.835969	0.850009



Graph-2

Case II:

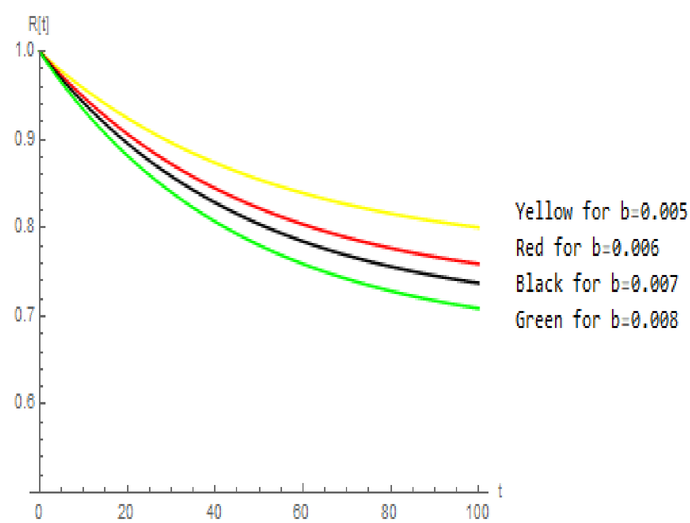
When a_5=0.1, a_6=0.01; demand of summer and winter products are different i.e. demand of winter products are greater than summer products.

Effect of failure rate on reliability of the system:

The reliability of the system is studied by varying their values as; b = 0.005, 0.006, 0.007, 0.008 and w =0.005, a_1 = 0.5, a_2 = 0.1, a_3 = 0.2, a_4 =0.9, a_5 = 0.1, a_6 =0.01.

Table 3: Effect of Failure rate on reliability of the system.

R(t) \ T	b= 005	b = .006	b = .007	b = .008
10	0.957931	0.948062	0.94163	0.933594
20	0.924168	0.906277	0.89564	0.881746
30	0.896413	0.872312	0.85851	0.840268
40	0.873585	0.844698	0.82852	0.807067
50	0.854806	0.822248	0.80429	0.780487
60	0.839359	0.803994	0.78472	0.759209
70	0.826653	0.789152	0.76891	0.742174
80	0.816200	0.777085	0.75614	0.728537
90	0.807602	0.767274	0.74582	0.717620
100	0.800529	0.759297	0.73749	0.708880



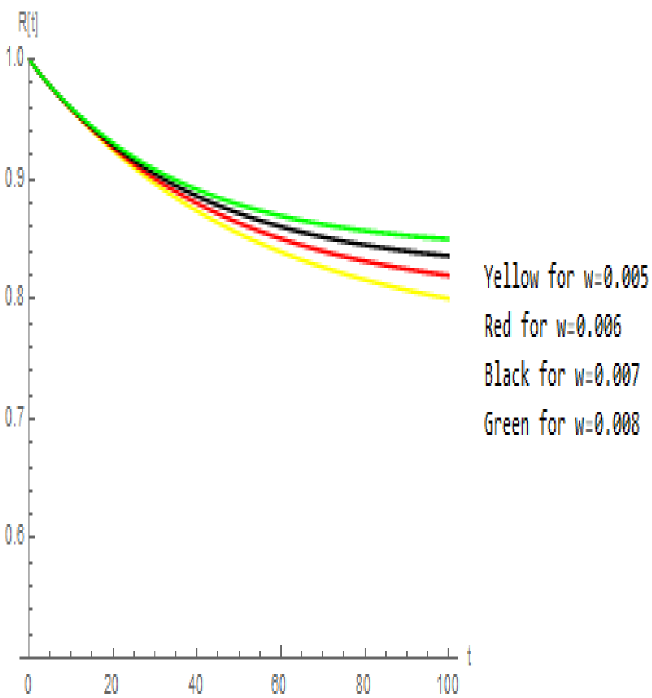
Graph 3

Effect of repair rate on reliability of the system:

The reliability of the system is studied by varying their values as; w = 0.005, 0.006, 0.007, 0.008 and b =0.005, a_1 =0.5, a_2 =0.1, a_3 =0.2, a_4 =0.9, a_5 =0.1, a_6 = 0.01.

R(t) T	w = .005	w = .006	w = .007	w = .008
10	0.957931	0.958544	0.959144	0.959733
20	0.924168	0.926271	0.928295	0.930242
30	0.896413	0.900527	0.904414	0.908088
40	0.873584	0.879978	0.885915	0.891434
50	0.854806	0.863574	0.871584	0.878914
60	0.839359	0.850479	0.860482	0.869500
70	0.826652	0.840024	0.851881	0.862423
80	0.816200	0.831679	0.845217	0.857102
90	0.807602	0.825017	0.840055	0.853101
100	0.800529	0.819699	0.836056	0.850094

Table 4: Effect of Repair rate on reliability of the system



Graph 4

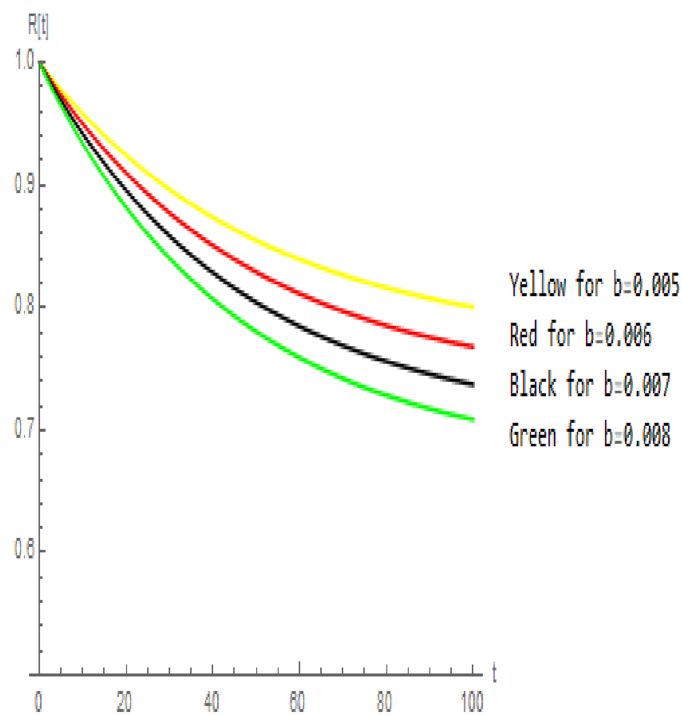
Case III:

When a_1=0.1, a_2=0.01; demand of summer and winter products are different i.e. demand of summer products are greater than winter products.

Effect of failure rate on reliability of the system: The reliability of the system is studied by varying their values as; b = 0.005, 0.006, 0.007, 0.008 and w =0.005, a_1 = 0.1, a_2 = 0.01, a_3 = 0.2, a_4 =0.9, a_5 = 0.5, a_6 =0.1.

R(t) T	b = .005	b = .006	b = .007	b = .008
10	0.9579312	0.949745	0.941632	0.933594
20	0.924168	0.909780	0.895641	0.881746
30	0.896413	0.877224	0.858513	0.840268
40	0.873585	0.850688	0.828523	0.807067
50	0.854806	0.829056	0.804296	0.780487
60	0.839359	0.811422	0.784725	0.759209
70	0.826653	0.797047	0.768915	0.742174
80	0.816200	0.785329	0.756143	0.728537
90	0.807602	0.775777	0.745825	0.717620
100	0.800529	0.767990	0.737490	0.708880

Table 5: Effect of Failure rate on reliability of the system.

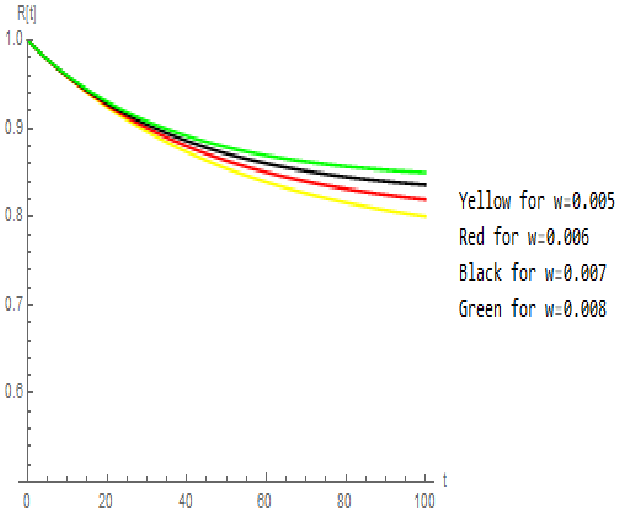


Graph 5

Effect of repair rate on reliability of the system:

The reliability of the system is studied by varying their values as; w = 0.005, 0.006, 0.007, 0.008 and b =0.005, a_1 =0.1, a_2 =0.01, a_3 =0.2, a_4 =0.9, a_5 =0.5, a_6 = 0.1.

Table6:Effect of repair rate on reliability of the system.



Graph 6

Discussion and Conclusion:

The tables show the variation of reliability with change in failure rates (b) and repair rates (w) and also observed this variation of demand and supply model for transient states.we observe the reliability of the system for three cases as demand vary due to variation in season. In case I we observe that the reliability of the system is greater than case II & case III i.e. when demand of summer and winter products are equal then system is more reliable than when demand of winter or summer product is greater than other. We also observed that reliability of the system decrease with increase in failure rate and reliability of the system increase with increase in repair rate, which is nearly 90% .The CAS software Mathematica enhance my work by reducing computation or calculation, time and give more accuracy in result, also give individual values of P0, P1, P2, P3, P4 at point t, which are continuous function of time not discrete.

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