ECCENTRIC BASED TOPOLOGICAL INDICES OF JAHANGIR GRAPHS

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Abstract: **For a given vertex u of $V(G)$of G, eccentricity $\varepsilon(u)$is the largest distance between u and any other vertex v of G. In the present paper, we have calculated the eccentric based topological indices of Jahangir graphs, $J_{n,m}$,$\forall m \geq 3$, $n \geq 1$**

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1 Introduction

Let $G = (V, E)$ be a graph, where V is a non-empty set of vertices and E is a set of edges. Suppose u and v are vertices of the graph G, we define their distance $d(u, v)$ as the length of the shortest path connecting u and v in G. For a given vertex u of $V(G)$ its eccentricity $\varepsilon(u)$ is the largest distance between u and any other vertex v of G. The maximum eccentricity over all vertices of G is called the diameter of G and denoted by $D(G)$ and the minimum eccentricity among the vertices of G is called radius of G and denoted by $R(G)$. The set of vertices whose eccentricity is equal to the radius of G is called the center of G. It is well known that each tree has either one or two vertices in its center. The molecular or chemical graph theory applies graph theory to mathematical modelling of molecular phenomenon, which is useful to learn molecular structure. This theory contributes a prominent role in the field of chemical sciences. The molecular structure is studied in terms of molecular descriptor known as topological indices. Topological indices are classified according how molecular graph is changed into a numerical value as, numerical values provided by adjacency matrix A or numerical value provided by distance matrix D, each of these again divided according to whether the matrix is processed by means of adding rows or column wise (leading to the vertex degree or to the distance sum) or by means of solving the matrix like determinants leading to polynomials or eigen values. In this paper we have surveyed eccentric based topological and calculated the same for graphs known as Jahangir graphs.

1.1 Survey on eccentric based topological indices

Definition : In 1997 Sharma, Goswami and Madan [1] initiated eccentric connectivity index, which is the distance and adjacency topological descriptor. The correlation coefficients obtained by them were ranging from 0.95 to 0.99 for eccentric connectivity index for various databases of physical properties of graph G. The said correlations were extremely greater than the first and well known topological index Wiener index. The eccentric connectivity index is defined as

$$\xi(G) = \sum_{u \in V(G)} d(u)\varepsilon(u)$$

It has been used for the build up of a range of mathematical models to guess biological activities of dissimilar nature. The mathematical properties of this index have been discussed by a number of authors in [2-5].

Definition : Analogous to other topological polynomials Alaeiyan, M., Mojarad, R. and Asadpour in [6,7] defined the eccentric connectivity polynomial is as, For a given graph G,

$$ECP\left(G,\lambda\right)=\sum_{u\in V(G)}d(u)\lambda^{\varepsilon(u)}$$

The association between the eccentric connectivity polynomial and the eccentric connectivity index is given by

$$ECP\left(G,\lambda\right)=\sum_{u\in V(G)}d(u)\lambda^{\varepsilon(u)}$$

$\xi(G)=ECP'(G,1)$ where $ECP^{'}\left(G,\lambda\right)$ is the first order derivative of $ECP\left(G,\lambda\right)$

Definition : Farooq, R. and Ali Malik [8] defined index by not considering the vertex degrees from eccentric connectivity and called it as total connectivity index. For a graph G we accomplish the total-eccentricity index as

$$\zeta\left(G\right)=\sum_{u\in V(G)}\varepsilon(u)$$

If G is k- regular graph then

$$\xi(u)=k\zeta\left(G\right)$$

Definition : Bukley et.al. [9] defined average eccentric connectivity index as

$$avec(G)=\frac{1}{n}\sum_{u\in V(G)}\varepsilon(u)$$

Definition : Ghorbani and Hosseinzadeh [10] Graovac et.al. [11] defined the Zagreb indices of a graph G in terms of eccentricity by replacing the degrees of the vertex with its eccentricities.

$$M_1^*(G)=\sum_{uv\in E(G)}\varepsilon(u)+\varepsilon(v)$$
$$M_1^{**}=\sum_{u\in V(G)}[\varepsilon(u)]^2$$
$$M_1^{**}=\sum_{u\in V(G)}[\varepsilon(u)]^2$$

2 Main results

2.1 Eccentric based topological indices of Jahangir graphs

In this segment, we consider the Jahangir graph structure and provide closed formulae of convinced topological indices for these graphs. Here, we find the analytically closed results of the eccentric connectivity index, eccentric connectivity polynomial, total eccentricity index, average eccentricity index and Zagreb eccentric indices for the graph $J_{n,m}$.

For $\forall m\geq 3$, Jahangir graph $J_{n,m}$ is a graph on nm + 1 vertices and m(n+1) edges. The Jahangir graph has diameter n+1 if n is odd and n+2 if n is even. For some even and odd number n vertex partition and edge partition of $J_{n,m}$ is given below

Vertex partition of $J_{n,m}$ for some even n

Table 1: Vertex partition of $J_{2,m}$ based on degree of vertex and eccentricity

$d(u)$	$\varepsilon(u)$	frequency
m	3	1
3	3	m
2	4	m

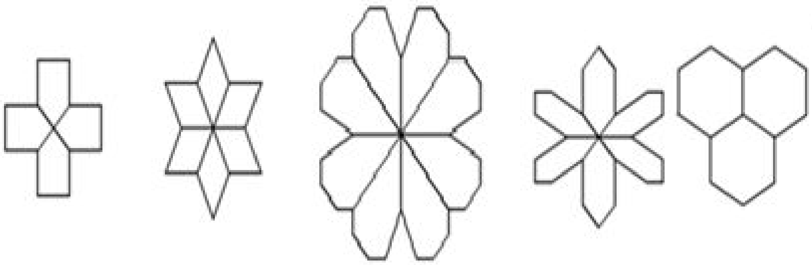

Figure. Jahangir graphs $J_{3,4}$, $J_{2,6}$, $J_{4,8}$, $J_{4,6}$ and $J_{4,3}$.

Table 2: Vertex partition of $J_{4,m}$ based on degree of vertex and eccentricity

$d(u)$	$\varepsilon(u)$	frequency
m	3	1
3	4	m
2	5	$2m$
2	6	m

Table 3: Vertex partition of $J_{6,m}$ based on degree of vertex and eccentricity

$d(u)$	$\varepsilon(u)$	frequency
m	4	1
3	5	m
2	6	$2m$
2	7	$2m$
2	8	m

Vertex partition of $J_{n,m}$ for some odd n

Table 4: Vertex partition of $J_{3,m}$ based on degree of vertex and eccentricity

$d(u)$	$\varepsilon(u)$	frequency
m	2	1
3	3	m
2	4	$2m$

Table 5: Vertex partition of $J_{5,m}$ based on degree of vertex and eccentricity

$d(u)$	$\varepsilon(u)$	frequency
m	3	1
3	4	m
2	5	$2m$
2	6	$2m$

Table 6: Vertex partition of $J_{7,m}$ based on degree of vertex and eccentricity

$d(u)$	$\varepsilon(u)$	frequency
m	4	1
3	5	m
2	6	$2m$
2	7	$2m$
2	8	$2m$

Edge partition of $J_{n,m}$ for some even n

Table 7: Edge partition of $J_{2,m}$ based on eccentricity

$(u,v) \in (\varepsilon(u), \varepsilon(v)$	frequency
(2,3)	m
(3,4)	$2m$

Table 8: Edge partition of $J_{4,m}$ based on eccentricity

$(u,v) \in (\varepsilon(u), \varepsilon(v)$	frequency
(3,4)	m
(4,5)	$2m$
(5,6)	$2m$

Table 9: Edge partition of $J_{6,m}$ based on eccentricity

$(u,v) \in (\varepsilon(u), \varepsilon(v)$	frequency
(4,5)	m
(5,6)	$2m$
(6,7)	$2m$
(7,8)	$2m$

Edge partition of $J_{n,m}$ for some odd n

Table 10: Edge partition of $J_{3,m}$ based on eccentricity

$(u, v) \in (\varepsilon(u), \varepsilon(v)$	frequency
(2,3)	m
(3,4)	$2m$
(4,4)	m

Table 11: Edge partition of $J_{5,m}$ based on eccentricity

$(u, v) \in (\varepsilon(u), \varepsilon(v)$	frequency
(3,4)	m
(4,5)	$2m$
(5,6)	$2m$
(6,6)	m

Table 12: Edge partition of $J_{7,m}$ based on eccentricity

$(u, v) \in (\varepsilon(u), \varepsilon(v)$	frequency
(4,5)	m
(5,6)	$2m$
(6,7)	$2m$
(7,8)	$2m$
(8,8)	m

Theorem 1: Let $J_{n,m}$, for all $n > 1$ be the Jahangir graph, then the eccentric polynomial is given by

$$ECP(J_{2n+1,m}, \lambda) = m\lambda^{n+1} + 3m\lambda^{n+2} + 2m\lambda^{n+2} \sum_{i=1}^{n} \lambda^i$$
$$ECP(J_{2n,_m}, \lambda) = m\lambda^{n+1} + 3m\lambda^{n+2} + 2m\lambda^{n+2} \sum_{i=1}^{n-1} \lambda^i + m\lambda^{2(n+1)}$$

Proof: Let $J_{n,m}$ be the Jahangir graph depending on n we have two cases when n is odd and even. If it is even then it is of the form $2n$ and for odd $2n$ + 1. By the definition of eccentric polynomial,

$$ECP\,(G, \lambda) = \sum_{u \in V(G)} d(u)\lambda^{\varepsilon(u)}$$

generalising the vertex partition of the J_{2n} using Table 1 to Table 3, and vertex partition of the J_{2n+1} using Table 4 to Table 6, we see that the Jahangir graph has only three kind of vertices. The central vertex is of degree m, m number of vertices of degree 3 and 2nmnumber of vertices of degree 2 in J_{2n+1} and $(2n - 1)m$ in J_{2n} .The eccentricity of central vertex is n + 1, degree 3 vertices have eccentricity n+ 2 and degree 3 vertices eccentricities ranges from n + 3 to $2n$ + 2 . Therefore we have

$$ECP(J_{2n+1,m}, \lambda) = m\lambda^{n+1} + 3m\lambda^{n+2} + 4m\lambda^{n+2} \sum_{i=1}^{n} \lambda^i$$
$$ECP(J_{2n,_m}, \lambda) = m\lambda^{n+1} + 3m\lambda^{n+2} + 4m\lambda^{n+2} \sum_{i=1}^{n-1} \lambda^i + m\lambda^{2(n+1)}$$

Theorem 2: Let $J_{n,m}$, for all $n > 1$ be the Jahangir graph, then the eccentric connectivity index is given by

$$\xi(J_{2n+1,m}) = m(6n^2 + 14n + 7)$$
$$\xi(J_{2n,_m}) = m(6n^2 + 10n + 3)$$

Proof: The formula for eccentric connectivity index is $\xi(G) = \sum\limits_{u \in V(G)} d(u)\varepsilon(u)$

And we know that, $\xi(G) = ECP'(G, 1)$

$$\begin{aligned}
ECP^{'}(J_{2n+1,m}, \lambda) &= \frac{d}{d\lambda}\left(m\lambda^{n+1} + 3m\lambda^{n+2} + 4m\lambda^{n+2}\sum_{i=1}^{n}\lambda^i\right)_{x=1} \\
&= m(n+1) + 3m(n+2) + 4m\sum_{i=1}^{n} n+2+i \\
&= mn + m + 3mn + 6m + 4m(\sum_{i=1}^{n} n+2 + \sum_{i=1}^{n} i) \\
&= 4mn + 7m + 4m(n(n+2) + \sum_{i=1}^{n} i) \\
&= 4mn + 7m + 4m\left(n(n+2) + \frac{n(n+1)}{2}\right) \\
&= 4mn + 7m + 4mn^2 + 8mn + 2mn^2 + 2mn \\
&= 14mn + 7m + 6mn^2 \\
&= m(6n^2 + 14n + 7)
\end{aligned}$$

and

$$\begin{aligned}
ECP^{'}(J_{2n,m}, \lambda) &= \frac{d}{d\lambda}\left(m\lambda^{n+1} + 3m\lambda^{n+2} + 4m\lambda^{n+2}\sum_{i=1}^{n-1}\lambda^i + 2m\lambda^{2n+2}\right)_{x=1} \\
&= m(n+1) + 3m(n+2) + 2m(2n+2) + 4m\sum_{i=1}^{n-1} n+2+i \\
&= mn + m + 3mn + 6m + 4mn + 4m + 4m(\sum_{i=1}^{n-1} n+2 + \sum_{i=1}^{n-1} i) \\
&= 8mn + 11m + 4m((n-1)(n+2) + \sum_{i=1}^{n-1} i) \\
&= 8mn + 11m + 4m\left((n-1)(n+2) + \frac{n(n-1)}{2}\right) \\
&= 8mn + 11m + 4m\left(n^2 + n - 2 + \frac{n^2-n}{2}\right) \\
&= 8mn + 11m + 4m\left(\frac{3n^2+n-4}{2}\right) \\
&= m(8n + 11 + 6n^2 + 2n - 8) \\
&= m(6n^2 + 10n + 3)
\end{aligned}$$

Theorem 3: Let $J_{n,m}$, for all $n > 1$ be the Jahangir graph, then the total eccentric connectivity index is

$$\zeta\left(J_{2n+1,m}\right) = 3mn^2 + 6mn + 2m + n + 1$$
$$\zeta\left(J_{2n,m}\right) == 3mn^2 + 4mn + n + 1$$

Proof : The Jahangir graph has only three kind of vertices with their degrees i.e. it has vertices of degree m, 2 and 3.The eccentricity of central vertex of degree m is n+ 1, $m$vertices of degree 3 have eccentricity n+ 2 and $2m$ degree 2 vertices eccentricities ranges from n+ 3 to 2n+ 2 in $J_{2n+1,m}$. Therefore

$$\begin{aligned}
\zeta\left(J_{2n+1,m}\right) &= 1\cdot(n+1) + m\cdot(n+2) + 2m\sum_{i=1}^{n} n+2+i \\
&= n + 1 + mn + 2m + 2m\left(n\left(n+2\right)\right) + 2m\sum_{i=1}^{n} i \\
&= n + 1 + mn + 2m + 2mn^2 + 4mn + 2m\cdot\frac{n(n+1)}{2} \\
&= 3mn^2 + 6mn + 2m + n + 1
\end{aligned}$$

In $J_{2n,m}$ the eccentricity of central vertex of degree m is n+ 1, m vertices of degree 3 have eccentricity n+ 2 and $2m$ degree 2 vertices of eccentricities ranges from n+ 3 to 2n- 1 and m degree 2 vertices with eccentricity 2n+ 2.

Therefore

$$\begin{aligned}\zeta\left(J_{2n,m}\right) &= 1\cdot(n+1)+m\cdot(n+2)+2m\sum_{i=1}^{n-1} n+2+i+m\cdot(2n+2)\\ &= n+1+mn+2m+2m\left((n-1)(n+2)\right)+\left(2m\sum_{i=1}^{n-1} i\right)+2mn+2m\\ &= n+1+mn+2m+2mn^2+2mn-4m+2m\cdot\tfrac{n(n-1)}{2}+2mn+2m\\ &= 3mn^2+4mn+n+1\end{aligned}$$

Theorem 4: Let $J_{n,m}$, for all $n > 1$ be the Jahangir graph, then the average eccentric connectivity is

$$\xi(J_{2n+1,m}) = \tfrac{1}{2nm+m+1}(6n^2+14n+7)$$
$$\xi(J_{2n,_m}) = \tfrac{1}{2nm}(6n^2+10n+3)$$

Proof: By the definition of average eccentricity connectivity index

$$avec(G) = \frac{1}{n}\sum_{u\in V(G)}\varepsilon(u)$$

where n is number of vertices Jahangir graph J_{2n+1}, m has $m(2n+1)$+1 vertices and $J_{2n,m}$ has $2nm$+1 vertices. Using the formula average eccentricity in Theorem 2: we have

$$\xi(J_{2n+1,m}) = \tfrac{1}{2nm+m+1}(6n^2+14n+7)$$
$$\xi(J_{2n,_m}) = \tfrac{1}{2nm}(6n^2+10n+3)$$

Theorem 5: Let $J_{n,m}$, for all $n > 1$ be the Jahangir graph, then the a first Zagreb index is

$$M_1^*(J_{2n+1,m}) = m\left[6n^2+14n+7\right]$$
$$M_1^*(J_{2n,m}) = m(6n^2+10n+3)$$

Proof : Generalising the edge partition of the J_{2n} using Table 7 to Table 9, and edge partition of the J_{2n+1} using Table 10 to Table 12

$$\begin{aligned}M_1^*(J_{2n+1,m}) &= \sum_{uv\in E(G)}\varepsilon(u)+\varepsilon(v)\\ &= m\left((n+1)+(n+2)\right)+m\left((2n+2)+(2n+2)\right)+2m\left((n+2)+(2n+2)\right)\\ &\quad +2m\cdot 2\sum_{i=0}^{n-1} n+2+i\\ &= m\left[(2n+3)+(4n+4)+(6n+8)+\left(4\sum_{i=0}^{n-1} n+2\right)+\left(4\sum_{i=0}^{n-1} i\right)\right]\\ &= m\left[12n+15+4(n-1)(n+2)+4\cdot\tfrac{n(n-1)}{2}\right]\\ &= m\left[6n^2+14n+7\right]\end{aligned}$$

$$\begin{aligned}M_1^*(J_{2n,m}) &= \sum_{uv\in E(G)}\varepsilon(u)+\varepsilon(v)\\ &= m\left((n+1)+(n+2)\right)+2m\left((n+2)+(2n+2)\right)+2m\cdot 2\sum_{i=0}^{n-1} n+2+i\\ &= m\left[(2n+3)+(6n+8)+\left(4\sum_{i=0}^{n-1} n+2\right)+\left(4\sum_{i=0}^{n-1} i\right)\right]\\ &= m\left[8n+11+4(n-1)(n+2)+4\cdot\tfrac{n(n-1)}{2}\right]\\ &= m\left[6n^2+10n+3\right]\end{aligned}$$

Theorem 6: Let $J_{n,m}$, for all $n > 1$ be the Jahangir graph, then the second Zagreb index is

$$M_1^{**}(J_{2n+1,m}) = \tfrac{1}{3}\left(14mn^3+30mn^2+38mn+3n^2+12m+6n+3\right)$$
$$M_1^{**}(J_{2n,m}) = \tfrac{1}{3}\left(5mn^3+21mn^2+55mn+36m+3n^2+6n+3\right)$$

Proof: The Jahangir graph has three class of vertices with their degrees i.e. it has vertices of degree m, 2 and 3. The eccentricity of central vertex of degree m is n + 1, m number of vertices of degree 3 have eccentricity n + 2 and $2m$ number of vertices of degree 2 have eccentricities ranging from n + 3 to $2n$ + 2 in $J_{2n+1,m}$. Therefore

$$
\begin{aligned}
M_1^{**}\left(J_{2n+1,m}\right) &= 1\cdot(n+1)^2 + m\cdot(n+2)^2 + 2m\sum_{i=1}^{n}\left(\underline{n+2}+i\right)^2 \\
&= n^2+2n+1+mn^2+4m+4mn+2m\left(\sum_{i=1}^{n}(n+2)^2\right)+2m\cdot 2(n+2)\left(\sum_{i=1}^{n}i\right)+2m\left(\sum_{i=1}^{n}i^2\right) \\
&= n^2+2n+1+mn^2+4m+4mn+2mn(n^2+4n+4)+(4mn+8m)\tfrac{n(n+1)}{2}+2m\tfrac{n(n+1)(2n+1)}{6} \\
&= \tfrac{1}{3}\left(14mn^3+30mn^2+38mn+3n^2+12m+6n+3\right)
\end{aligned}
$$

In $J_{2n,m}$, the eccentricity of central vertex of degree m is n+ 1, m number of vertices of degree 3 have eccentricity n+ 2 and $2m$ number of degree vertices 2 of eccentricities ranging from n+ 3 to $2n$-1 and m degree 2 vertices with eccentricity $2n$+ 2.

Therefore

$$
\begin{aligned}
M_1^{**}\left(J_{2n,m}\right) &= 1\cdot(n+1)^2 + m\cdot(n+2)^2 + 2m\sum_{i=1}^{n-1}\left(\underline{n+2}+i\right)^2 + m\cdot(2n+2)^2 \\
&= n^2+2n+1+mn^2+4m+4mn+2m\left(\sum_{i=1}^{n-1}(n+2)^2\right)+2m\cdot(n+2)\left(\sum_{i=1}^{n-1}i\right)+2m\left(\sum_{i=1}^{n-1}i^2\right) \\
&\quad +4m(n^2+2n+1) \\
&= n^2+2n+1+5mn^2+8m+12mn+2m(n-1)(n+2)^2+2m\cdot(n+2)\tfrac{n(n-1)}{2} \\
&\quad +2m\tfrac{n(n-1)(2n-1)}{6} \\
&= \tfrac{1}{3}\left(5mn^3+21mn^2+55mn+36m+3n^2+6n+3\right)
\end{aligned}
$$

Theorem 7: Let $J_{n,m}$, for all $n > 1$ be the Jahangir graph, then the third Zagreb index is

$$
\begin{aligned}
M_2^{*}(J_{2n,m}) &= \tfrac{m}{3}\left(14n^3+39n^2+31n+6\right) \\
M_2^{*}(J_{2n+1,m}) &= \tfrac{m}{3}\left(14n^3+43n^2+39n+10\right)
\end{aligned}
$$

Proof: Generalising the edge partition of the J_{2n} using Table 7 to Table 9 and edge partition of the J_{2n+1} using Table 10 to Table 12.

$$
\begin{aligned}
M_2^{*}(J_{2n,m}) &= \sum_{uv\in E(G)}\varepsilon(u)\varepsilon(v) \\
&= m\left((n+1)(n+2)\right)+2m\sum_{i=0}^{n-1}(n+2+i)(n+3+i) \\
&= m\left[n^2+3n+2+2\sum_{i=0}^{n-1}(n+2)(n+3)+2\sum_{i=0}^{n-1}(2n+5)i+2\sum_{i=0}^{n-1}i^2\right] \\
&= m\left[n^2+3n+2+2n(n^2+5n+6)+\left(2(2n+5)\sum_{i=0}^{n-1}i\right)+\left(2\sum_{i=0}^{n-1}i^2\right)\right] \\
&= m\left[n^2+3n+2+2n^3+10n^2+12n+2(2n+5)\tfrac{n(n-1)}{2}+2\tfrac{n(n-1)(2n-1)}{6}\right] \\
&= \tfrac{m}{3}\left(14n^3+39n^2+31n+6\right)
\end{aligned}
$$

$$
\begin{aligned}
M_2^{*}(J_{2n+1,m}) &= \sum_{uv\in E(G)}\varepsilon(u)\varepsilon(v) \\
&= m\left((n+1)(n+2)\right)+2m\sum_{i=0}^{n-1}(n+2+i)(n+3+i)+m(2n+2)(2n+2) \\
&= m\left[n^2+3n+2+2\sum_{i=0}^{n-1}(n+2)(n+3)+4(n+1)^2+2\sum_{i=0}^{n-1}(2n+5)i+2\sum_{i=0}^{n-1}i^2\right] \\
&= m\left[n^2+3n+2+2n(n^2+5n+6)+4(n^2+2n+1)+\left(2(2n+5)\sum_{i=0}^{n-1}i\right)+\left(2\sum_{i=0}^{n-1}i^2\right)\right] \\
&= m\left[n^2+3n+2+2n^3+10n^2+12n++4n^2+8n+4+2(2n+5)\tfrac{n(n-1)}{2}+2\tfrac{n(n-1)(2n-1)}{6}\right] \\
&= \tfrac{m}{3}\left(14n^3+43n^2+39n+10\right)
\end{aligned}
$$

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